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From bilateral trade to centralized markets: A search model for commodity exchanges in Africa[☆]

Yaw Nyarko[a,b], Heitor S. Pellegrina[b,*]

[a] NYU New York, United States
[b] NYU Abu Dhabi, United Arab Emirates

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ABSTRACT

Several African countries have recently centralized their agricultural markets by launching a commodity exchange. What will be the impact of such a move? Who will be the winners and the losers? We develop a simple search model to study the impact of introducing a commodity exchange in a village economy where traders and farmers exchange on a bilateral basis. We study the efficiency gains from moving from the status quo to a trading regime where farmers have the option of selling their produce to a commodity exchange. We describe how the gains from trade are distributed between farmers, traders and the commodity exchange itself. We show that a dual economy where high-cost farmers remain in the bilateral exchange market while low-cost ones sell to the commodity exchange can exist in equilibrium, and that forcing all farmers to sell into the commodity exchange can make some farmers worse off.

1. Introduction

The absence of modern trading institutions is perceived as an important cause of the large costs of trading in developing countries.[1] In most African countries, in particular, agricultural markets are still decentralized: farmers and traders search for a trading partner in local markets to trade on a bilateral basis. This trading environment, however, is expected to change in the near future. As shown in Fig. 1, a few African countries have recently launched a commodity exchange and many are planning to follow in the next decade. In contrast to the decentralized system, in a market governed by a commodity exchange, transactions between farmers and traders occur in a predetermined location and are typically mediated by market makers who could be thought of as the Walrasian auctioneer as used in standard economics discourses.

Motivated by these recent developments, there has been a growing number of papers and policy reports discussing the effects of commodity exchange markets on price fluctuations and price convergence between African regions.[2] There has been, however, comparatively less progress in terms of understanding how commodity exchange markets will affect the transactions between farmers and traders. In this paper, we formulate a model to study the impact of introducing a commodity exchange in a village economy where farmers and traders exchange on a bilateral basis. In the context of our model, we address the following questions: How does a commodity exchange affect market efficiency? How are the gains from trade distributed between farmers, traders, and the commodity exchange itself? Which types of farmer benefit from the commodity exchange? Under what conditions do decentralized markets co-exist with the commodity exchange?

We start our analysis by making several observations about the market structure of rural villages of African economics. We illustrate our observations with a study area in Ghana and complement our description with data from other sub-Saharan countries. In this context, farmers establish commercial partnerships with traders to whom they sell their produce on a regular basis. Traders, in their turn, sell farmers' produce to downstream markets. Traders are typically small, itinerant,

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* Corresponding author.
E-mail addresses: yaw.nyarko@nyu.edu (Y. Nyarko), heitor.pellegrina@nyu.edu (H.S. Pellegrina).

[1] See Mezui et al. (2013), Rashid et al. (2010) and Atkin and Donaldson (2015) for an analysis of trade costs in Africa.

[2] Several papers have recently studied the impact of commodity exchanges on the co-movement of prices across regions (Hernandez et al., 2015; Andersson et al., 2017; Minten et al., 2014, 2017; Gelaw et al., 2017; Meijerink et al., 2014; Sehgal et al., 2012; Tenderere and Gumbo, 2013). Also, see Mezui et al. (2013) and Rashid et al. (2010) for policy reports documenting the experience of developing countries with the implementation of commodity exchange markets.

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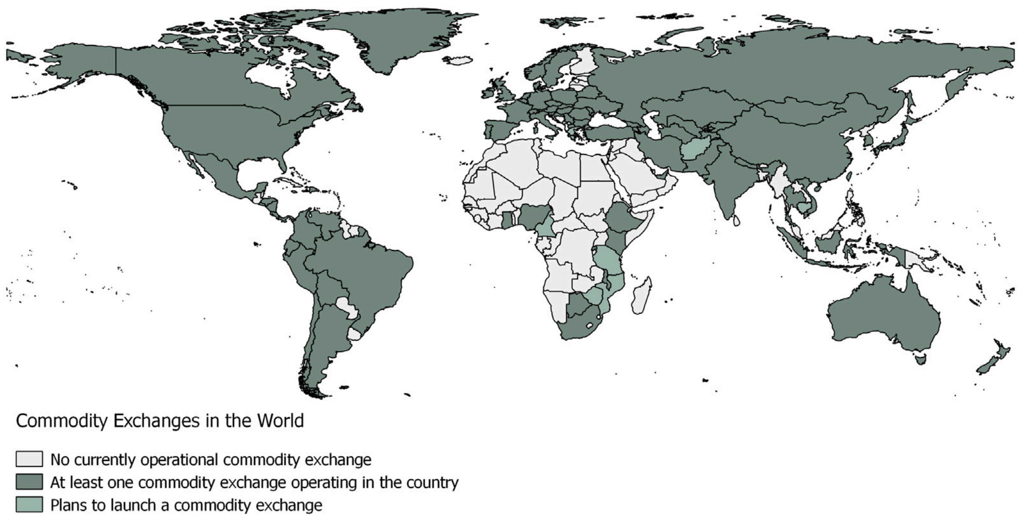

Fig. 1. Commodity exchanges in the world as of 2019.

and constrained in terms of the number of farmers with whom they can trade in any given day or week. Since they can partner with a limited number of farmers, they tend to be selective when they pick a farmer to form a partnership. Because they lack information about which farmers have not yet sold their produce, searching for farmers takes time and resources. When farmers do not find a buyer for their produce, they often incur some post-harvest losses.

To reduce the risks of post-harvest losses that are inherent in this trading environment and guarantee sales to farmers, Ghana has progressively introduced a commodity exchange market since 2018, called the Ghana Commodity Exchange (GCX). We describe how this commodity exchange operates, the contracts that brokers trade, the fees charged by the commodity exchange to farmers who wish to sell their produce there, among other characteristics.

Informed by our empirical observations, we develop a parsimonious model in which establishing commercial partnerships between small traders and smallholder farmers is a time-consuming activity. We consider a dynamic economy where farmers have heterogeneous transport costs and traders face a homogeneous price at which they sell produce in downstream markets. In every period, a trader can pay a fixed cost to search for a farmer. If a trader pays this search cost, she is matched to a farmer and the trade cost of the farmer is revealed to her. If a trader finds a farmer, she and the farmer engage in bargaining to determine the price at which she buys one unit of an agricultural good. If they reach an agreement, they form a partnership that is carried over to future periods until an exogenous shock breaks their partnership. If there is no agreement (or no bargaining at all), the trader has to pay the search cost again to find another farmer in future periods. If farmers are not matched to a trader by the end of a period, they incur post-harvest losses.

After presenting this search and bargaining environment, which we refer to as the bilateral exchange market (hereafter, BEM), we use a fixed point argument to prove the existence of an equilibrium and to show that it is unique. We then derive the aggregate supply of agricultural produce and show that there exist two sources of inefficiency in this economy. The first one comes from the randomness of the search process. In every period a mass of farmers who could potentially generate positive market surplus—i.e., whose trade costs are below the price of a unit of the agricultural good in the downstream market—are not matched to any trader due to the randomness of the search process. The second one comes from traders who strategically reject bargaining with high-cost farmers. Specifically, we show that there exists a mass of high-cost farmers whose transportation costs are below the price of produce in downstream markets and could generate a positive market surplus, but who are still rejected by traders who opt to forgo current gains for the chance of being matched with low-cost farmers in future periods.

We next give farmers the option of selling their produce to a commodity exchange market (hereafter, CEM) instead of only having the option of waiting to be matched with traders. If farmers choose to sell to the CEM, they guarantee their sales and avoid the risk of post-harvest losses, but then they have to pay a fee that creates a wedge between the price of their produce in the CEM and the price of produce in downstream markets. We find that the traditional BEM tends to coexist with the CEM, as some farmers choose to sell to the CEM and others to the BEM. In addition, we demonstrate that the benefits of the CEM are larger for low-cost farmers. It is therefore these low-cost farmers who opt to sell their produce to the CEM. High-cost farmers are still left behind, as they opt to wait for traders to sell their produce in the BEM.

We find that the CEM generates welfare benefits to some but not all farmers. Some low-cost farmers who choose to sell to the CEM are better off, since they avoid the risk of post-harvest losses. As these farmers move to the CEM, some traders may lose because they have to operate in a BEM where the remaining farmers have higher costs. Some high-cost farmers who stay in the BEM are potentially also better off. Even though they remain in the BEM, they are less likely to be strategically rejected by traders, and they also could have a higher bargaining power when negotiating prices—that is because traders have fewer farmers competing for their attention. Still, there is a fraction of farmers who could generate a positive market surplus who remain without options to sell their produce.

We show that aggregate welfare rises upon the introduction of the CEM due to the elimination of post-harvest losses. We indicate how the CEM affects the bargaining and negotiations with the lower numbers of high cost farmers, potentially increasing the willingness of traders to accept higher prices, and expanding the set of (high cost) farmers who are matched and therefore sell their produce.

We use our model to examine two types of implementations of commodity exchanges that have been widely discussed by policymakers: the full and the partial mandate. In partial mandate, as in the new Ghana Commodity Exchange, farmers have the option of selling either to the CEM or to the BEM, which is the case in which both types of regimes might coexist. In contrast to this system, we also analyze the full mandate, as in the Ethiopian Commodity Exchange. In that case, the government bans the BEM and forces all farmers to sell to the CEM, which guarantees a minimum volume of transactions in the

CEM.[3] We find that the full mandate system can expand the reach of the CEM in terms of farmers, increasing the revenues of the commodity exchange, but at the welfare cost of some farmers and still leaving behind a fraction of high-cost farmers without options to sell their produce. Hence, through our model, we show how the incentives for the formation of informal and illegal markets emerge, an important concern among policymakers.[4]

Lastly, we study the economic implications of risk-aversion, showing that the CEM can provide a consumption smoothing instrument to farmers. In addition, we discuss the robustness of our analytical results to different simplifying assumptions. Specifically, we examine the importance of the number of matches a trader can have, the exogeneity of the search intensity of traders, the sources of price volatility in our model, and the entry conditions of traders.

Related literature. Our paper relates to different strands of research. First, our model predicts the co-existence of the CEM with decentralized markets, which relates to the concept of dual markets that has a long history in development economics. As elegantly and forcefully noted by Gollin (2014), it was the core of the analysis of Arthur Lewis, considered by many as the father of development economics, in his seminal paper Lewis (1954). Arthur Lewis emphasized a dual economy where there was a modern sector living side by side with a traditional sector. In our paper we may think of the commodity exchange market as the modern and the Bilateral Exchange Market as the traditional. Our main theorem shows how the two can live side by side in equilibrium.

Within the more recent literature, bargaining models have been used in papers looking at the impact of price information, usually transmitted via mobile phones, given to farmers who sell their goods to traders, including Hildebrandt et al. (2015), Aker (2010, 2008), Courtois and Subervie (2014) and Svensson and Yanagizawa (2009). We contribute to this literature by examining how the introduction of a commodity exchange affects the transaction price between farmers and traders.

Our theoretical framework builds on two early models of decentralized markets, Rubinstein and Wolinsky (1985, 1987). In Rubinstein and Wolinsky (1985), the authors formulate a search and bargaining model with two sided markets. Their model has two types of agents: buyers and sellers. Agents in each side of the market are homogeneous. There is no strategic rejection and, as in our paper, matching probabilities are fixed. In Rubinstein and Wolinsky (1987), the authors develop a search and bargaining model with three types of agents: buyers, sellers and middlemen. Agents in each side of the market are again homogeneous and matching probabilities are fixed in steady state. In our paper, we use tools developed in these two papers—particularly on the Nash Bargaining outcome. We have essentially two sided markets, farmers and traders. (We could think of a third type of agent, the final buyer, however this is trivial as the price in downstream markets is fixed). In contrast to these papers, we have strategic rejection, heterogeneous agents in one side of the market, and the coexistence of decentralized and Walrasian markets.

By applying the tools from these earlier models, our paper relates to applications of search models to asset markets. A key reference here is Duffie et al. (2005), who develop a search and bargaining model for over-the-counter markets. They have investors with and without assets, and with low and high costs of holding assets. Investors with high costs of holding assets search for investors with low costs to buy their assets and vice versa. Lagos and Rocheteau (2009) and Afonso and Lagos (2015) extend the model in Duffie et al. (2005) by allowing for more flexible asset position. In these papers, when agents are matched they bargain over the price of an asset and there is no strategic rejection. In our model, agents instead bargain over a contract that includes multiple transactions over time and there is strategic rejection.

The interactions between farmers and traders in our model also relates to applications of search models to labor markets, including Pissarides (1985) and Mortensen and Pissarides (1994). In these two papers, workers and firms search for each other and, when matched, they bargain over wages.[5] To the best of our knowledge, this literature has not modeled the co-existence of Walrasian and decentralized markets, in part because Walrasian markets are extremely rare in labor markets. We believe that our framework has applicability for cases where Walrasian markets can coexist with decentralized ones, particularly for markets where agents negotiate loans of assets such as machinery and warehouses.

We know of few papers that study the co-existence of Walrasian and decentralized markets. One reference is Rust and Hall (2003) who introduce a Walrasian market into the middlemen model formulated by Spulber (1996). They have three types of heterogeneous agents in their model, buyers, sellers and middlemen. In Rust and Hall (2003), there is no bargaining and prices are posted ex-ante. (See Rogerson et al. (2005) for an overview of models with ex-post and ex-ante price setting.) Another reference is Miao (2006), who studies the introduction of a commodity exchange in a search an bargaining environment. Different from his paper, we have strategic rejection and we study the distributional gains from trade. A common feature in these two papers is the existence of a constant flow of new agents who leave the market after a single transaction. In our model, we instead have a fixed mass of agents on each side of the market (some matched and others not) who bargain over long-term contracts. In addition, while these papers consider agents with risk-neutral preferences, which is the common approach in this literature, we examine the case in which farmers are risk-averse.[6]

Lastly, we contribute to recent research in trade using tools from search theory (Antras and Costinot, 2011; Allen, 2014; Startz, 2016; Antràs and Staiger, 2012; Krishna and Sheveleva, 2017; Atkin and Donaldson, 2015). Closest to our paper is Antras and Costinot (2011), who examine the gains from trade between countries in a dynamic search model where traders and farmers establish contracts to exchange agricultural commodities. In their model, farmers are homogeneous and can produce one of two goods. In our model, we have a single good, but farmers have heterogeneous trade costs.[7] In addition, while in their framework farmers can only sell their produce to traders, here we study the effects of giving farmers the option of selling their produce to a commodity exchange. Another recent paper in this area is Atkin and Donaldson (2015), who show that imperfect pass-throughs contain information about market structure and provide estimates of

[3] A common risk that commodity exchanges face is the lack of sufficient transaction volumes. In that case, the capacity that the commodity exchange has to guarantee the delivery of a product is limited. Furthermore, commodity exchanges have large fixed costs, but low marginal costs of individual transactions. If there is not enough volume transacted on the floor, commodity exchanges may not generate sufficient revenues to pay for their fixed costs. The risks of insufficient scale are higher when a commodity exchange coexists with a decentralized market, which is the more common system. In some cases, to minimize this risk, governments opt for a fully mandated system, where the government bans some types of market transactions from taking place beyond the commodity exchange floor.

[4] See more detailed discussions about the tradeoffs between partial and full mandate systems in Anderson and Baulch (2017).

[5] See Rogerson et al. (2005) for a throughout review of the literature on the applications of search models to labor markets. Another common approach in this literature is to have search without bargaining. In this case, workers and firms post wages ex-ante and workers search until they find a firm that offers a wage that is above their reservation wage.

[6] Another reference is Gehrig (1993) who formulates a static model with random match and bargaining. Since his model is not dynamic is does not account for the strategic considerations that we examine in our model.

[7] Allen (2014) and Krishna and Sheveleva (2017) also formulate search models to analyze the transactions of goods in developing countries, but different from Antras and Costinot (2011), their models are based on ex-ante posting of prices and no-bargaining between the two sides of a market.

imperfect-pass-throughs of manufacturing goods in Ethiopia and Nigeria. In the agricultural context, Fafchamps and Hill (2008) document imperfect pass-throughs for coffee in Uganda. Here, we model a specific micro-economic mechanism that generates imperfect pass-throughs: the search costs of decentralized agricultural markets.[8]

2. Case study in a rural African market

This section describes the *status quo* farming environment and market structure of an agricultural market of a village in a typical sub-Saharan African country. We document the market structure in our study area based on research we carried in Ghana from 2015 through 2019.[9] We complement our description using data from the World Bank for other sub-Saharan countries. We also describe key characteristics of the commodity exchange recently introduced in Ghana, the Ghana Commodity Exchange (GCX). We close this section with a brief discussion of how we map some of our observations to the structure of the model that we develop in Section 3.

2.1. Our study area

We will focus on the crops which are traded on the Ghanaian commodity exchange: primarily maize, soya and rice, with some sesame and sorghum. Our focus will be on smallholder farmers in Ghana. They form the bulk of the farming in the country, both in terms of the numbers of people involved and in terms of the volume produced.

We begin with general observations about the market microstructure in these areas. What we describe here is the *status quo* environment before any commodity exchange is introduced. Our pilot study area is a portion of the central part of Ghana, in the Kumawu Traditional area (the Sekyere Kumawu and Sekyere Afram Plains parliamentary districts plus small amounts of 2 or 3 others surrounding these districts). This area covers around 5000 square kilometers, approximately 2% of the land mass of Ghana. As of the most recent publicly available census, our pilot study area has around 120,000 inhabitants, making it a relatively sparsely populated area.

2.2. Main observations

Below we list key observations about the study area. The next section describes the key agents in the market.

1. **Land.** For smallholder farmers, land issues are not currently a major constraint on their production. When such farmers seek to expand their production at the margin, there is typically vacant land within their properties or informal markets to rent their neighbors' piece. But land can sometimes be an issue for large-scale farming.
2. **Labor.** Farmers use their own time and labor on their farms and also hire laborers. The labor is required to clear (or "weed") the farms and also to carry produce from the interior of the farm to the farm gate. As of the time of writing, it costs around GHS 20 (around US $2) per day for these laborers, who in the local parlance are called "by day laborers." These by day laborers help with cutting the weeds, harvesting or spraying.[10]
3. **Transport cost.** The transport sector involves high fees for moving produce for farmers, relative to their revenues. Yet those with the produce can have them transported to local markets for the most part. These fees are commonly perceived by farmers as surmountable so long as they find customers to sell their produce to.
4. **Agricultural inputs.** Fertilizer use is extremely low. Farmers indicate to us that they know that fertilizer use is important, but they claim that it does not make economic sense to invest in fertilizers. Some farmers are afraid of spending money on fertilizers, perhaps with borrowed money, only to see the markets collapse on them at harvest time. Other farmers complain that they have liquidity or cash constraints which prevent them from purchasing fertilizers. Those farmers also do not go to the banks for loans because, again, they fear the consequences of a market collapse at harvest time when they have no money to repay their loans.
5. **Technology.** Advanced technology is non-existent and given current prices, the use of such technologies is probably not optimal at this time and at the scale of production of the farmers. There are no irrigation schemes among the smallholder farmers we worked with. In a national survey we conducted among 1200 farmers, only one group hired the services of a tractor. The vast majority of farmers use only one implement in their farming, the cutlass.
6. **Finance.** Many farmers indicated that with more capital they could expand their farms. When asked why they did not go to the bank for a loan, they frequently say that this is because of fear of not getting a good price for their output and then falling into debt. Farmers often take loans from traders in exchange for selling their goods to the trader at harvest time, but they claim to dislike this arrangement. This is because traders dictate a price to them when the harvest came, thereby extracting a high implicit interest rate on the loan.
7. **Demand.** Lack of sustainable demand for farmers' crops seems to be the biggest constraint to the development of the smallholder agricultural sector. Farmers complain a lot about not being able to get buyers for their produce. When farmers are asked why they do not use fertilizer or advanced technology or take bank loans, the answer almost always seems to involve the lack of sustained markets for their goods.
8. **Storage.** Warehouses and storage facilities are non-existent for many crops of many farmers. There are a variety of techniques that farmers employ which amount to implicit storage. For example, yam farmers keep the yams in the ground until they are ready to sell. Other crops are left unmatured and treated with chemicals to make them flower quickly when there is a need to sell these. Still, despite farmers' efforts to minimize potential losses, there are large post-harvest losses in the region.

2.3. Key agents in the market

Farmers. The main crops grown by farmers in our area are yam, plantain, cassava and maize. Some farmers also grow cocoa, but cocoa is a cash crop that is managed by the government. Some less important crops, by volumes and revenues, include garden eggs (eggplant), tomatoes and other vegetables, cocoyam, groundnut and rice. Very few of the farmers use any kind of mechanization. The main implement used by farmers is the cutlass and nothing else. The cutlass is used to clear weeds, make holes in the ground to insert seeds, etc. Farmers do pay attention to the seeds and the methods of planting. They obtain seeds from the previous harvest or through nurseries in neighboring communities. They get a lot of advice from the government Ministry of Food and Agriculture (MoFA) extension agents. Farmers use chemicals, namely weedicide, to keep out unwanted grass and shrub. Some farmers complained about pests affecting their crops. They also complained that

[8] In our model, we assume perfect competition in downstream markets where traders resell their produce, but search markets in upstream markets where traders buy their produce from farmers. This market structure is consistent with results from Fafchamps and Hill (2008), who document a large pass-through between international and wholesale prices, but a small pass-through between international and farmers' price in Uganda.

[9] Nyarko is grateful to the International Growth Center and Anonymous donors for the research grants that enabled this research to take place.

[10] An alternative method of contracting labor is by acreage, which costs approximately GHS 150 (around US $30) per acre. The laborer given that contract will be required to work on that area to get paid and will be paid proportionately to the total acreage worked on.

because of insufficient funds they are unable to engage in pest control and use herbicides. Farmers indicate that they are cash-constrained and almost never purchase the required amount of fertilizers as instructed by the government extension agents. When they "get some good money they will invest in fertilizers", they told us, otherwise, they take their chances on the over-worked soil on their farms.[11]

In our study area, as in many sub-Saharan African nations, farmers live on their tribal lands and are often from families that have been there for centuries. Geographic mobility is not that high among farmers growing the crops we study. There is a great difference in transport cost between being close to the main road or far from one, or being relatively close to one but having no access to that road. Even a few miles distance away from the main road through dense forest and steep hills or difficult river crossings can make a huge difference. Different crops are grown in different areas. However, within an area that, for example, grows maize, there could be a lot of variation in production costs due to the reasons just mentioned. This difference would exist even when the geographic distances are relatively small and agroclimatic conditions are similar.

Traders. There are many different types of traders. Most of these are women. They are intermediaries of various sizes, but most of them are very small. When we formulate our model, these small traders are the ones that we have in mind. Many of the small traders take goods from farmers and send them to regional markets which are around 1 to 2 h away by car. These small traders are the majority of those who live in our study area. Our farmers occasionally interact with big traders who collect goods from farmers for sale in Accra. One farmer mentioned, "I trade with 2 people from Accra and I sell to them on Thursdays." One interesting feature of traders' activities caught our attention. Sometimes traders "buy the farm," as they say in the local parlance. What this means is that the farmer and trader negotiate for a certain amount of the farmer's farm—for example, 2 acres of a farmer's plantain farm. The trader will pay the farmer a price and then the trader will be responsible for hiring the laborers to harvest the produce (the plantain in this example) and to take the produce from the farm to the village. This is a common way in which the farmers deal with their lack of liquidity or their lack of ability to pay upfront for their labor and transportation costs.

The trading activities of small itinerant traders are often, but not always, coordinated by a "market queen" (Clark, 1994), who is usually an elected representative of traders in regional market for a given staple.[12] Market queens coordinate the market space: for example, they determine how many trucks can enter in the urban area and who can become an itinerant trader in the area; they also settle disputes and negotiate with local governments (Lyon, 2000). Another barrier that traders face to enter in a market is that they require many years of experience to consolidate their activities. Traders typically build long-standing relationships with farmers. These relationships operate as informal contracts based on farmer's ability to deliver the produce in time and trader's ability to sell their produce in central markets. Despite those barriers, there is a large number of very small itinerant traders operating in these markets.[13]

Traders and local markets. One interesting feature of markets caught our attention and may be a local response to the various constraints faced by market participants. In any one market town, markets operate once a week. The days of the week differ in different towns. For example, the market town Bodomase operates on Fridays and the market in Juaben on Wednesdays. By having markets open once a week, traders are able to aggregate produce from more farmers and get the volume needed to make their operations scale up. All traders would, for example, converge on Bodomase on Fridays. The farmers in that area will farm most of the week, then collect all their produce on Thursday night at their house or in a local storage area and have them ready for traders to inspect and hopefully purchase early in the morning on Friday. In some local markets, we did hear of price fixing by traders. We were told of some instances where traders agree in advance on what the price of a particular crop should be. We did not hear of this for all crops, and this effect seemed to be dying down. One farmer, a woman, said to us "they used to have fixed prices for tomatoes, however that process has died.". We did not find much of this price fixing occurring in the big cities and towns.

Despite small traders' efforts to scale up their operations, they tend to be cash constrained, which limits how much they can buy in advance from farmers before weekly markets. In addition, transportation technologies deter how much they can bring to local markets. In our study area, most traders rely on small vehicles. In other sub-Saharan regions, transportation technologies can be substantially worse. For example, in Ethiopia and Tanzania, more than 80% of farmers have their produce transported on foot, on a bicycle or with a draft animal, and less than 18% of farmers have their produce transported on a truck, bus or a minibus (see Appendix Table A.1).

We did not document in our research buyers who buy for their own consumption (like the poultry farmers who purchase maize)—they are similar to the final buyers in regional markets that traders sell to. Almost all of the buyers in our study area resell their produce in retail markets, therefore we consider them all to them as traders.

Traders, information asymmetries and search. In general, farmers do not know the price of goods in major markets and do not arbitrage price differentials across time. One question we posed was this: why don't the farmers just call a friend in the market in the main city to ask for the current prices? We found out in research that the farmers did not have friends who had access to the market prices. Since prices moved around so much, even if they knew someone in the city, it would probably not be worth to pay that person to go to the market every week just to check prices.

Modern storage methods are too expensive for farmers to afford, which constraints their ability to make strategic decisions about the timing of their sales. Data for other sub-Saharan regions indicate similar constraints in Ethiopia, Tanzania and Malawi. Appendix Table A.1 shows that more than 80% of farmers in these countries use traditional storage methods. When asked about the reason why they store their produce, the majority of farmers declare that they store for their own consumption, and only 3 to 8 percent of them say that they store to capture higher prices in the future.

Farmer-trader matches and post-harvest losses. Farmers typically wait for traders to buy their produce at a good price and do not sell their produce in local markets themselves.[14] They told us often that they

[11] Some farmers indicated that animals destroy their farms. None of the farms we studied had any fences or barriers demarcating and sealing off their farms. The region is currently being invaded by cows roaming the bush led by itinerant or nomadic pastoralists. These pastoralists travel from the Sahel areas, particularly those affected by climate change. They head south to less affected areas. This has resulted in many violent clashes recently between the owners of these roaming cowherds and the indigenous people farming on the same contested lands. These issues have been documented in our study area and in surrounding areas.

[12] The phenomenon of the market queen is a common feature of agricultural markets in West Africa. Clark (2018) documents the existence of market queens in Togo and Ghana. Achebe (2020) describes market queens in Senegal, Ghana and Togo. In East Africa, other types of significant barriers to the entry of traders have been documented in Bergquist and Dinerstein (2020).

[13] Lyon (2000) documents the many ways through which these relationships are built. For example, through gifts, attending funerals and traveling to villages to participate in the trading of produce.

[14] This is the case in other sub-Saharan regions. As discussed in Fafchamps and Hill (2005), almost all of coffee growers in Uganda sell their produce at the farm-gate.

would be at their farm gate looking for or waiting for traders but would not have any who visit them. These farmers often live in faraway and remote areas where trade costs are high. Similarly, we spoke to many traders who told us that it is hard to find farmers to trade with. In particular, it seemed as if there could be viable matches if only the traders wanting goods and farmers with the goods to sell could locate each other. We also remark here that we found many situations where the farmers would negotiate with traders on the appropriate price to sell their goods at and not reach an agreement.

2.4. The Ghana Commodity Exchange (GCX)

According to the Oxford Dictionary of Economics, a commodity market is "A place or institution through which commodities are traded. Markets were originally places or buildings, where traders could come together, which facilitated comparisons of price and quality. [...] Commodity markets include both spot markets, where goods are traded for immediate delivery, and forward and futures markets, where prices are agreed in advance for delivery at various dates in the future." In November of 2018, Ghana launched the first commodity exchange market in West Africa, called the Ghana Commodity Exchange (GCX), to operate as a spot market and trade crops from various locations.[15] In its inauguration speech, the president of Ghana Nana Akufo declared that the goal of GCX is to reduce post-harvest losses and benefit farmers by securing storage for their harvest.

A commodity to be traded in the GCX is a contract that is indexed by a crop (e.g., maize, or soya), a quality grade (grade 1 is the best grade, grade 4 is the worst), and a warehouse location (e.g., in Tamale or Sandema in the north of Ghana). For example, there could be trade in White Maize grade 1 sitting at the Tamale warehouse — this contract would be given the symbol TAWM1, where TA stands for Tamale, the WM stands for White Maize, and 1 stands for grade 1. There are currently 9 warehouses, all across the country.

At fixed times during the day, e.g., Monday at 1pm GMT, there will be trade in a contract, e.g., TAWM1. Sell side brokers who represent farmers with maize (TAWM1) to sell will post offers on the market. Buy side brokers who represent people who want to buy the maize (e.g., poultry farmers) will similarly post ask prices on the platform. The system then matches buyers and sellers with compatible prices and trade takes place.[16] Once this is done, the trade is complete. The buy side broker pays for the good by transferring money into what is called the Central Depository, which is run by the exchange. The Central Depository in turn will transfer that money to the broker on the sell side who then transfers the money to the farmer who deposited the crop in the first place.

GCX is owned by the national government and any farmer may pay a fee and sell their produce at one of the official warehouses. These official warehouses are located in regional centers, close to the location of regional markets. Therefore, the costs of transporting farmers' good to the nearest warehouse tend to be similar to the cost of taking a good to a regional market. The fees are charged to cover the operation costs of the commodity exchange and are proportional to the volume of goods deposited by the farmer in the warehouse. The basic fee amounts to up to 10% of the price of the good. Adding drying, fumigation and cleaning increases this fee up to 40%. For the 40 or so farmers we interviewed in our pilot study, the average fee paid to the commodity exchange was approximately 17%. Appendix Section F includes a few receipts that we have collected of actual transactions between farmers and the GCX for reference.[17]

Lastly, due to the technical nature of the work, brokers in the commodity exchange need a minimum of Bachelor's degree. They are required to have experience in Financial Management and IT. There is a lot of technical information that they need to grasp, and very quickly. They need computer skills, business management, understanding business models and trading trends, and risk management. The small itinerant traders operating in the traditional markers are, therefore, unlikely to become brokers in the commodity exchange, at least in the short run.

2.5. From our observations to the model

Our observations indicate that search costs in our study area of Kumawu are substantial. The lack of information about farmers forces traders to spend a substantial amount of their time to the activity of trading itself, such as contacting suppliers and inspecting farms. Once traders find farmers, they negotiate the price of the produce, since there is not a common pre-established price in their market. Traders are generally small and, due to cash-constraints and poor transportation technologies, cannot buy from several farmers in any given week or day. Because search takes time, many farmers are not matched to a trader and end up using their produce for subsistence consumption or incur post-harvest losses. In general, farmers do not to sell themselves their produce in downstream markets.

In the next section, we model the transactions between small traders and smallholder farmers who are currently producing corn and yam, since these are the transactions that will be directly affected by the GCX.[18] This commodity exchange market will give farmers the option of selling their produce to the CEM at any time, avoiding the risks of post-harvest losses if they are not found by any trader. As highlighted by the president of Ghana in his speech during the inauguration of the GCX: "Ghanaian farmers will gain access to secured storage for their harvest and good warehousing management practices, thereby improving their take-home sales". Our model will be designed to capture precisely this type of loss.[19]

For tractability, we adopt a few simplifying assumptions to focus on this particular view of the benefits of the commodity exchange. Specifically, we adopt two assumptions about farming production based on our empirical observations. First, given that the scale of production is extremely small and that there is little to no use of agricultural inputs among smallholder farmers, we assume that land is the only production factor. Therefore, our results should be interpreted with caution as there are many ways in which they may understate the positive impact of the commodity exchange. For example, while in

[15] The only derivative-based exchange in Africa is the South African Futures Exchange (SAFEX).

[16] Acceptable price pairs are those where the buy side broker's bid price exceeds the sell side brokers ask price. After a match the system generates an acceptable price, the midpoint. In practice, however, buy side brokers look at the price list and post and accept prices of sell side brokers that they find agreeable.

[17] We remark, following Lewis (1954), that we still have cheap labor in the traditional rural sector and high cost labor in the modern urban sector. The high level of the fees of the CEM are due in part to having the staff with formal education, buildings, warehouses, etc. These fees are high relative to incomes that smallholder farmers get with their sales. The fees are therefore high in a relative sense. The CEM is currently not for profit and the fees cover only operating costs of the exchange. We also notice that the CEM is still in its early stage of implementation, and the high fees reflect in part this condition. With time, as the training costs of brokers drop and the CEM expands, such fees are expected to drop.

[18] Larger farms producing cocoa, for example, already sell their produce to the government or directly to international trading companies.

[19] In addition, the president declared that "Most often, there are no formal contractual agreements in place, resulting in trade disputes between our farmers and buyers which undermine our marketing system. These are some of the challenges we are aiming to address." In our model, we will be capturing these types of informal contractual agreements between farmers and traders by assuming that they establish trading partnerships that endure for multiple periods of time.

the short run technologies may be fixed at the current low levels of adoption, in the long run a CEM might encourage the adoption of new technologies. Second, we do not model credit markets. Our observations suggest that the low uptake of loans is partially due to the uncertainty generated by the lack of consistent demand, as could be obtained from a centralized market. With the implementation of a commodity exchange, farmers might increase their loan uptake—although there are certainly different views on this future potential of the exchange (see Jayne et al. (2014)).

3. The Bilateral Exchange Market (BEM)

This section develops a parsimonious search and bargaining model to describe an agricultural market such as the one represented by our study area. This model will help us understand the impact of the introduction of a commodity exchange which is beginning to take place in this community. We organize this section in three parts. First, we characterize the economy at the *status quo* when there is no commodity exchange, which we call the pure bilateral exchange market (BEM). Second, we define the equilibrium and prove the existence and uniqueness. Third, we discuss the gains from trade and the aggregate supply of agricultural produce in the community. In the next section, we introduce a commodity exchange market into the economy.

3.1. Economic environment in a pure BEM

Consider an economy with two types of agents, farmers (F) and traders (T). This economy operates over time. The time dimension is discrete. Farmers and traders live forever. In each period, farmers produce one unit of a non-storable agricultural good. They can sell their agricultural good to a trader who then take their produce to a regional market, where agricultural goods are sold at a price P. (We often refer to P as the "Accra" price.) We denote by c the costs that farmers incur in every period that they take the agricultural good from their farms (which are usually in remote areas) to the farm gate (roadside) or to local markets. We can think of these as transport costs from the farm (which could be in the "bush" to the road), although it is easy to think of c also incorporating other production costs. There is a unit mass of homogeneous traders and a unit mass of farmers with costs coming from a uniform distribution between 0 and c^{max}, such that the density of farmers equals to $g(c) = 1/c^{max}$. We normalize to 0 the price that a farmer receives for his produce if they do not sell it to a trader. We consider this to be a post-harvest loss. We think of a scenario where the farmer has an abundance of food for subsistence (home consumption) and the crops we are studying are primarily for sale, with those for home consumption coming from their gardens or easily obtained on an almost daily basis from their farms.[20]

In this economy, farmers and traders can either be unmatched or be matched in a trading partnership with each other. Traders who are interested in forming a new trading partnership can pay an upfront cost κ, in every period they are searching. Traders need to find farmers who are "real" and reliable, since legal enforcement mechanisms are weak and fraud is a real possibility. Farmers, on their turn, focus on production and do not search for traders. When a trader finds an unmatched farmer, the cost parameter of the farmer c is revealed to the trader who then decides whether to negotiate the price of produce $p(c)$. Farmers are matched with a trader with exogenous probability μ^F. Traders are matched with a farmer with exogenous probability μ^T.[21] If they reach an agreement, they leave the network and start a trading partnership that is exogenously broken with a probability β. Once the farmer leaves the network, she is replaced with another farmer with the same cost. (This replacement guarantees that traders' and farmers' decisions are replicated in every period.) In this environment, search is costly for traders for two reasons: first, because they have to pay an upfront cost of κ to search; second, because if they do not find a farmer during the search process, or if they do find a farmer but choose to reject negotiation, they have to wait until the next period to search again.

We note that the cost c borne by farmers is incurred each time the farmer's goods are sent to the farmgate to the trader, but that it is not incurred if there is no match with a trader. Also, the trader cost κ is incurred in each period the trader is unmatched and searches for a new farmer; this cost is not incurred if the trader is matched with a farmer.

When farmers and traders make their decisions, they consider the option of waiting to find a better trading partner in future periods. Specifically, their gains from trade is the value of establishing a partnership and being matched for multiple periods against the value of being unmatched and waiting for better trading partners. For a farmer, the cost of waiting is to lose the opportunity of selling his or her produce to a trader by $p(c)$. For a trader, the cost of waiting is to lose the opportunity of selling an agricultural good right in the beginning of the next season at price P.

Let V^{FM} and V^{FU} be the values of being matched and unmatched for a farmer, and V^{TM} and V^{TU} the respective values for a trader. The value functions of farmers and traders are given by the following Bellman equations:

$$V^{FM}(c) = u(p(c) - c) + \delta\left\{\beta V^{FU}(c) + (1-\beta)V^{FM}(c)\right\} \quad (1)$$

$$V^{TM}(c) = \max\left\{P - p(c) + \delta\left\{(1-\beta)V^{TM}(c) + \beta V^{TU}\right\}, V^{TU}\right\} \quad (2)$$

$$V^{FU}(c) = \delta\left\{\mu^F V^{FM}(c) + (1-\mu^F)V^{FU}(c)\right\} \quad (3)$$

$$V^{TU} = \max\left\{\delta\left\{\mu^T \int V^{TM}(c)g(c)dc + (1-\mu^T)V^{TU}\right\} - \kappa, 0\right\}, \quad (4)$$

The first equation describes a farmer with cost c who is matched with a trader in the beginning of the period. It says that the value function of the matched farmer with cost c is equal to what the farmer gets in the first or current period, the price $p(c)$ minus the cost c, plus the discounted value of the future utility value. The future value is determined by whether the farmer is unmatched or matched in the next period, $V^{FU}(c)$ or $V^{FM}(c)$, events that occur with probabilities β and $(1-\beta)$ respectively.[22] In what follows, we present our proofs based on the case in which farmers are risk-neutral (linear utility), i.e., the case in which $u(p(c) - c) = p(c) - c$. This greatly simplifies our exposition, but still communicates the essential steps that we use in our proofs for the more general case in which farmers are risk-averse. (Appendix Section D presents the full proof for the risk-averse case.)

[20] Following the approach adopted in the literature on market microstructure and search, including as Rust and Hall (2003), Miao (2006), Antras and Costinot (2011), among many others, we simplify agents' production choices as to focus on the consequences of search costs to their market choices.

[21] In a typical search model, the probabilities μ^F and μ^T are determined by a matching function—which generally adopts a constant returns to scale form—that depends on the mass of unemployed farmers and traders in the market. Here, assuming that μ^F and μ^T simplifies substantially our analysis. In the appendix, we provide additional justification to our approach.

[22] Notice that we impose a one period delay between production and trade. We allow this delay to capture the fact that there are seasons and a natural rhythm to the farming cycle. Here, we think of each period as a whole cycle of a crop. During the planting and growing seasons, farmers meet with traders and negotiate prices to guarantee their sales when the harvest season arrives. When farmers are done harvesting and cleaning their crops, they hand their produce to traders. Many farmers sell all their crops in one go. Even if it is not in one go, it is over a very short period of time.

Later, in Section 4.7, we discuss in detail the economic implications of risk-aversion.[23]

The second Eq. (2) pertains to a trader who is matched with a farmer with cost c in the beginning of the period. That trader has to decide whether to trade with that farmer (the left-hand term in the bracket after the max) or else to walk away (we call this strategic rejection) and become unmatched, with a value function of V^{TU}. If the trader does trade with the matched farmer, then the trader makes in the current period a profit equal to the difference between the big city price P and the bargained price $p(c)$ plus the discounted value of the expected return to being matched with a farmer of cost c.

The third and fourth equations, (3) and (4), pertain to the unmatched farmer and unmatched trader. In each case they receive 0 in the current period, their future returns are discounted by δ and they receive the expected return to being matched and unmatched in the next period, events that take place with probabilities μ^F and $1-\mu^F$ for the farmer and μ^T and $1-\mu^T$ for the trader. The trader pays a search cost of κ when unmatched and beginning a search.

Let η denote the bargaining power[24] of the trader and define $\phi = (1-\eta)/\eta$ as the power of a farmer relative to a trader. In particular, at each value of c where there is trade between the farmer and the trader, we have the relation that the surplus going to the farmer is ϕ times the surplus going to the trader:

$$V^{FM}(c) - V^{FU}(c) = \phi\left\{V^{TM}(c) - V^{TU}\right\}. \tag{5}$$

We stress here that the equation above will be required to hold only for those values of c such that both parties, the farmer and the trader, want to trade.[25] In particular, this equation will only be required to hold when the maximum on the right-hand side of Eq. (2) and also of Eq. (4) each occurs in the first term and not in the second term inside the maximum operator. If we consider as parameters c^{max}, P, c, β, δ, κ, μ^F, μ^T, and ϕ the Eqs. (1)–(5) are a system of 5 equations in the 5 unknowns $V^{FM}(c)$, $V^{TM}(c)$, $V^{FU}(c)$, V^{TU}, and $p(c)$, which suggests that in principle we can get solutions for the 5 unknowns in terms of the parameters.

We next define the market equilibrium in this economic environment. The timing of events captures the idea that farmers first meet traders to negotiate with a price in mind, without carrying their produce with them. Once there is an agreement, then the produce needs to be taken from the farm, incurring a cost c. As such, the cost c is not a sunk cost in the price negotiation between the two of them.

3.2. Definition of equilibrium in pure BEM with positive flows

Definition 1. Fix a model with parameter set $\Omega = \{c^{max}, P, c, \beta, \delta, \kappa, \mu^F, \mu^T, \phi\}$ and cost parameters c uniformly distributed in the set $[0, c^{max}]$. The equilibrium is a pricing function $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$, $V^{TU}(c)$, V^{TU}, defined via the following two stage game.

1. In the first stage, a pricing function $p(c)$ is set within the BEM. Conditional on the behavior of farmers described below, no individual trader has an incentive to offer a price different from $p(c)$ to a farmer with cost function c.
2. In the second stage, each farmer with cost c decides whether to trade or not, upon being matched with a trader. Farmers trade whenever $p(c) > c$, they are indifferent between trading or not when $p(c) = c$ and we suppose that no farmer chooses to trade when $p(c) < 0$. In addition, whenever trade occurs, we impose the Nash Bargaining solution (5).[26] We assume that farmers only accept a price equal to or better than that from the Nash Bargaining solution.[27]
3. The value function Eqs. (1)–(4) hold whenever there is (positive) flow in the BEM meaning that the participation constraint given by (2) and (4) hold for some c (i.e., the max occurs in the first term of the right-hand side of (2) and (4) for some values of c).

3.3. Proof of existence of equilibrium

Our method to prove the existence of equilibrium is to sequentially use Eqs. (1)–(5) to prove the existence of $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$, $V^{FM}(c)$, V^{TU} which satisfy Eqs. (1)–(5). We use Eqs. (1)–(3) and (5) iteratively to eliminate $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$ and $V^{FM}(c)$ in Eq. (4) so that Eq. (4) becomes a function only of V^{TU}. We then show that this final version of Eq. (4) admits a fixed point or solution, V^{TU*}. The equilibrium value for V^{TU} then generates the equilibrium values of $p(c)$, $V^{TM}(c)$, $V^{FU}(c)$ and $V^{TU}(c)$ by re-tracing the initial substitutions. Here, we describe the main steps of the proof, relegating details to Appendix Section C.

Making V^{TU} a function of parameters. In what follows, we define the following parameters to ease our exposition:

$$\sigma_1 = \phi(1-\delta) - \delta(1-\beta) + \beta\delta\phi + \delta\mu^F\phi + 1;$$

$$\sigma_2 = \sigma_1 - \mu^F\phi.$$

Claim 1. *σ_1 and σ_2 are positive.*

Proof. In the appendix. □

We start by using Eqs. (1) and (3) to obtain expressions for $V^{FU}(c)$ and $V^{FM}(c)$ as a function of $p(c)$:

$$V^{FM}(c) = \frac{(p(c)-c)\left(1-\delta+\delta\mu^F\right)}{(1-\delta)\left(1-\delta+\beta\delta+\delta\mu^F\right)}, \tag{6}$$

and

$$V^{FU}(c) = \frac{(p(c)-c)\,\delta\mu^F}{(1-\delta)\left(1-\delta+\beta\delta+\delta\mu^F\right)}. \tag{7}$$

Using the expressions above to eliminate $V^{FM}(c)$ and $V^{FU}(c)$ from Eq. (5), we obtain an expression for $p(c)$ as a function of c, $V^{TM}(c)$ and V^{TU}:

$$p(c) = c + \phi\left(1-\delta+\beta\delta+\delta\mu^F\right)(V^{TM}(c) - V^{TU}). \tag{8}$$

We use the equation above to eliminate $p(c)$ from $V^{FU}(c)$ in (7):

$$V^{FU}(c) = \frac{\delta\mu^F\phi\left(V^{TM}(c) - V^{TU}\right)}{1-\delta}. \tag{9}$$

[23] We notice that, for the linear utility case, we prove existence and uniqueness of equilibrium both in the pure BEM and also in dual markets when there exists a CEM. For the concave utility case, we prove existence and uniqueness in the pure BEM, but we only show existence in the CEM. See details in Appendix Section D.

[24] η is the Nash bargaining weight so that any surplus (less outside options) is shared in the proportions η for the trader and (1-η) for the farmer.

[25] Notice that in Eq. (5), even though the bargaining parameter ϕ is fixed, bargaining outcomes are still a function of outside options and so are "endogenous". The outside options define the size of the "gains from trade" that needs to be divided among the players or our bargaining game—the trader and the farmer. After the gains from trade have been defined by the outside options, the split of the gains is defined typically by the different time discount rates in models like that of Rubinstein and Wolinsky (1985). In particular, the final negotiated price between traders and farmers in our case is the outcome of a bargaining process in which farmers are actively choosing whether to leave the negotiation or not.

[26] This is in keeping with the literature, where, as in this paper, the microfoundations of the Nash Bargaining rule process are not explicitly described.

[27] For example the trader could offer a take it or leave it offer to the farmer for a very small amount over the farmer's cost c, and the farmer could be modeled as being forced to take that offer. Instead we think of some within period match-specific game which justifies the Nash Bargaining posited here.

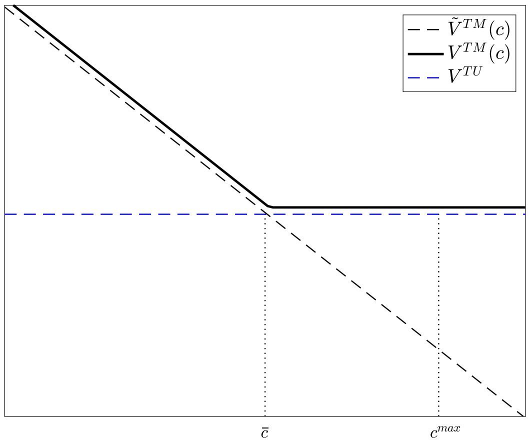

Fig. 2. The value function of matched traders.

We now move to expression (2). We will substitute the equations that we constructed so far into Eq. (2) to obtain an expression of $V^{TM}(c)$ as a function of c and V^{TU}. To do so, we construct an equation for $V^{TM}(c)$ that will be a solution to the fixed point problem defined in expression (2) when the left-hand side of the maximum operator is larger than the right-hand side.

First, let $V^{TM}_{RHS}(c)$ be the first term of the right-hand side of Eq. (2): $V^{TM}_{RHS}(c) \equiv P - p(c) + \delta((1-\beta)V^{TM}(c) + \beta V^{TU})$. Replace $p(c)$ in this expression with Eq. (8) to get:

$$V^{TM}_{RHS}(c) = P - c - V^{TM}(c)\left(\sigma_1 - 1\right) + V^{TU}\left(\sigma_1 + \delta - 1\right). \quad (10)$$

Compute now the equation that we obtain when the left-hand side of Eq. (2) equals the first term of the maximum operator on the right-hand side of the equation, i.e., when we have $V^{TM}(c) = V^{TM}_{RHS}(c)$. Isolate $V^{TM}(c)$ in this expression and define its solution (or fixed point) to be $\tilde{V}^{TM}(c)$:

$$\tilde{V}^{TM}(c) = \frac{\sigma_1 + \delta - 1}{\sigma_1} V^{TU} + \frac{P - c}{\sigma_1}. \quad (11)$$

Note that $\tilde{V}^{TM}(c)$ is linear and decreasing with respect to c with a slope of $-1/\sigma_1$. Fig. 2 illustrates the shape of $\tilde{V}^{TM}(c)$. Using this definition for $\tilde{V}^{TM}(c)$, the next claim derives an expression for $V^{TM}(c)$ as a function of V^{TU} and c.

Claim 2. *Fix a V^{TU} and all the other parameters of the model, including c. Then $V^{TM}(c)$ is a solution to Eq. (2) if and only if $V^{TM}(c)$ is given by*

$$V^{TM}(c) = \max\left\{\tilde{V}^{TM}(c), V^{TU}\right\}. \quad (12)$$

Proof. In the appendix. □

Let $\bar{c}$ denote the value of c when $\tilde{V}^{TM}(c)$ equals V^{TU}, then:

$$\bar{c} = P - (1-\delta)V^{TU}. \quad (13)$$

Here, when $V^{TU} = 0$, $\bar{c} = P$. Fig. 2 plots $V^{TM}(c)$ and $\bar{c}$. It shows that the utility value of being matched for a trader drops with farmers' cost c. This utility value equals $\bar{c}$ when traders are indifferent between being matched or unmatched. The value V^{TU} thus sets a lower bound to the utility $V^{TM}(c)$ that traders can obtain by trading with a farmer. We notice that search costs will influence the strategic behavior of traders indirectly via the equilibrium value of V^{TU}. Appendix Section E studies the comparative statics of the model when we change these search costs.

Now replace $V^{TM}(c)$ in Eq. (9) by the new expression for $V^{TM}(c)$ obtained from Eqs. (11) and (12) to obtain:

$$V^{FU}(c) = \frac{\delta\mu^F\phi\left(P - c - (1-\delta)V^{TU}\right)}{(1-\delta)\sigma_1} \text{ for } c < \bar{c} \quad (14)$$

and $V^{FU}(c) = 0$ for $c \geq \bar{c}$.

Since $V^{FU}(c)$ in Eq. (14) is a function only of V^{TU} and parameters, we now have expressions for all of our equilibrium variables $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$ as functions of V^{TU}. By construction these expressions satisfy Eqs. (1)–(5) except for (4). Hence, once we find an equilibrium value of V^{TU} satisfying Eq. (4), we can plug that into the just mentioned functions to generate equilibrium values of $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$. This is our next step. All expressions from now until the end of the proof will be functions of V^{TU} and the parameters of the model and otherwise independent of $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$. To get an equation of V^{TU} as a function of the exogenous parameters, we have to solve a fixed point problem, which we do next.

Defining and solving the fixed point problem for V^{TU}. To define the fixed point problem that we use to find the solution for V^{TU}, we work with expression (4). First, define

$$IV^{TM} = \int_0^{\bar{c}} \tilde{V}^{TM}(c)g(c)dc + \int_{\bar{c}}^{c^{max}} V^{TU}g(c)dc. \quad (15)$$

Next, define the first term on the right-hand side of expression (4) to be V^{TU}_{RHS}:

$$V^{TU}_{RHS} = \delta(\mu^T IV^{TM} + (1-\mu^T)V^{TU}) - \kappa. \quad (16)$$

It is easy to check that V^{TU}_{RHS} is a quadratic function of V^{TU} by inserting the equation we constructed for $\tilde{V}^{TM}(c)$ into IV^{TM} and integrating over the relevant values of c. Let a_{quad} be the coefficient of the quadratic term and a_{const} be the constant term (i.e., the value of V^{TU}_{RHS} when $V^{TU} = 0$). Simple algebra shows:

$$a_{quad} = \frac{\delta\mu^T(1-\delta)^2}{2c^{max}\sigma_1} \quad (17)$$

and

$$a_{const} = \kappa^{BEM}_{max} - \kappa \quad (18)$$

where

$$\kappa^{BEM}_{max} = \delta\mu^T \frac{P^2}{2c^{max}\sigma_1}. \quad (19)$$

Since $\sigma_1 > 0$, then $\kappa^{BEM}_{max} > 0$. As we shall see below, κ^{BEM}_{max} represents the maximum value of κ such that we have an equilibrium with positive trade flows.

Our next claim shows key properties V^{TU}_{RHS} that we use in our solution. Fig. 3 illustrates these properties and highlights the fixed point problem that we have, In particular, the point at which V^{TU}_{RHS} crosses the 45 degree line gives the equilibrium value of V^{TU}.[28] Note that κ^{BEM}_{max} is the maximum value of κ such that V^{TU}_{RHS} crosses V^{TU}.

Claim 3. *The function V^{TU}_{RHS} has the following properties*

1. V^{TU}_{RHS} is convex in V^{TU};

[28] While our goal is not to provide a quantitative evaluation of the CEM, the parameter values that we pick for our figures are inspired by our institutional setting. We set the price of produce to 40 throughout, which is a common value in GHS of a bag of corn of 50 kg during the 2015–2018 period. We assume that $c^{max} = P$ so that, in principle, all farmers in a catchment area can generate positive market surplus if they sell their produce to traders. We use $\delta = 1/1.05$, which is a typical value in the literature and assumes a interest rate of 5% for the economy. We set μ^F to 0.2 and μ^T to 0.8 to capture the fact that there is an abundance of farmers per trader. Since we do not have a strong prior to the bargaining parameter ϕ, we set this value to 1, which gives an equal share of the gains from trade to traders and farmers. Given that there is recurrent new matches and broken matches according to our interviews, we set $\beta = 0.5$. In some figures, we deviate from these values to make our argument more salient. Appendix Section E provides a more detailed discussion of these parameters based on comparative statics of the model.

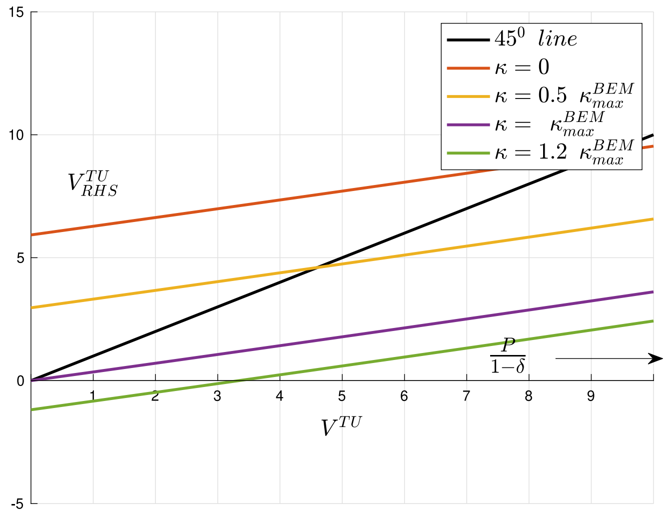

Fig. 3. The fixed point problem of V^{TU} for different values of κ in a BEM. Notes: This figure illustrates the fixed point problem that we solve. The point in which V^{TU}_{RHS} crosses the 45 degree line defines a solution to our problem. We constructed this figure based on the following parameter set $[c^{max}, P, \beta, \delta, \mu^F, \mu^T, \phi] = [40, 40, 0.25, 0.5, 0.2, 0.8, 1]$.

2. if $\kappa < \kappa^{BEM}_{max}$, then $V^{TU} - V^{TU}_{RHS} > 0$ at $V^{TU} = 0$, and, if $\kappa > \kappa^{BEM}_{max}$, then $V^{TU} - V^{TU}_{RHS} < 0$ at $V^{TU} = 0$;
3. at $V^{TU} = \frac{P}{1-\delta}$, $V^{TU} - V^{TU}_{RHS} = P + \kappa > 0$.

Proof. (1) It follows from $a_{quad} > 0$. (2) When $V^{TU} = 0$, $V^{TU}_{RHS} = a_{const}$ so part (2) follows immediately from Eq. (19). (3) Set $V^{TU} = P/(1-\delta)$ in V^{TU}_{RHS}, then simple algebra proves this point. □

Proposition 1. *For all $\kappa < \kappa^{BEM}_{max}$, there exists a unique equilibrium to the BEM model. When $\kappa > \kappa^{BEM}_{max}$ the equilibrium involves no trader making visits to farms.*

Proof. See details in Appendix A. □

Strategic rejection. The last proposition concludes our proof. It shows that there exists an equilibrium with positive trade flows between farmers as long as the fixed costs for searching is not excessively high ($\kappa < \kappa^{BEM}_{max}$). In this equilibrium, there is a range of farmers with costs above $\bar{c}$ and below P who would generate positive gains from trade in the short-run, but who traders nevertheless reject in order to wait for the opportunity of being matched with low-cost farmers. We call this phenomenon "strategic rejection".

3.4. Gains from trade and aggregate supply in a pure BEM

We start this section with the distributional gains from trade. We then turn to the aggregate supply of this economy. For the latter, we have to track the matched and the unmatched farmers and determine their mass, which we left aside in the exposition of the model so far.

Gains from trade in a pure bem. Fig. 4(a) shows the price function and how the gains from trade are distributed between farmers and traders in the pure BEM.[29] It shows that farmers with costs above $\bar{c}$ and below P are rejected by traders, which generates a loss of potential matches equal to the area denoted by G relative to the optimum without search frictions. All farmers below $\bar{c}$ sell their produce at the bargained price $p(c)$. Area A represents the surplus captured by farmers when matched and area B the surplus of traders.[30]

Tracking matched and unmatched farmers in steady state. Fix a period t. Then fix any c and let m_t be the probability that a farmer of type c is matched. Alternatively, we can think of there being a unit mass of farmers with cost c and of m_t as being the fraction of those farmers who are matched. Let u_t be the mass of farmers of cost c who are unmatched in period t such that $m_t + u_t = 1$. Since in each period the matched become unmatched with probability β and the unmatched become matched with probability μ^F, the mass of matched farmers of any cost c in period $t+1$ is

$$m_{t+1} = \mu^F u_t + (1-\beta) m_t. \tag{20}$$

and the mass of unmatched farmers is

$$u_{t+1} = (1-\mu^F) u_t + \beta m_t. \tag{21}$$

We look for steady state values m^* and u^* of m_t and u_t, defined to be the situation where $m_{t+1} = m_t = m^*$ and $u_{t+1} = u_t = u^*$. Simple algebra shows that the steady state values of m and u to be

$$u^* = \frac{\beta}{\beta + \mu^F} \tag{22}$$

and

$$m^* = \frac{\mu^F}{\beta + \mu^F}. \tag{23}$$

In steady state, a fraction m^* of farmers of type c are matched (or will have a probability m^* of being matched) and a fraction $u^* = 1 - m^*$ are unmatched.

In the earlier sections we ignored the distinction between the matched and unmatched and instead assumed that there was some density function $g(c)$ of the relevant farmers which both traders and farmers use in computing their value functions (1)–(5). The relevant farmers used in those value function computations are those in the set of unmatched farmers. Suppose that the economy wide density function for all farmers, matched or unmatched, of different values of c is uniform on $[0, c^{max}]$, whose density function is $\frac{1}{c^{max}}$. Then the density function representing the unmatched farmers is

$$g(c) = \frac{u^*}{c^{max}}. \tag{24}$$

with u^* given by Eq. (22).[31] We shall use this formulation in computing the supply function, gains from trade and in comparing welfare in the BEM model just described, as well as in the dual economy which we develop in Section 4.

Aggregate supply in a pure bem. We now characterize the aggregate supply of the economy in steady state. The active farmers in the economy are those with cost between 0 and $\bar{c}(P)$. Only matched farmers will produce output in a given period, and, as argued above, the mass of those active farmers is m^*. Since each farmer produces one unit of output, the mass of the matched farmers is equal to the mass of output

[29] In the figure, farmers with cost $c > \bar{c}$ receive an offer of $p(c) = c$ from traders. To see how this is an equilibrium according to our definition, notice that traders do not have incentives to offer a price $p(c) > p$, since then farmers would accept their offer and traders prefer to reject any trade with such farmers, nor a price $p(c) < c$, since then traders would make negative profits. In addition, for $p(c) = c$, farmers are indifferent between accepting traders' offers or not and, for simplicity, we assume that they prefer not to sell their produce.

[30] In our analysis, the bargained price, $p(c)$, is increasing in the cost parameter c. One may object by saying that in the developing country context those with higher values of c should have weaker bargaining positions and therefore lower prices. Appendix Section B explains why one should not expect this to be the case. We also point out here that the "profits" of farmers, that is, the difference between price and cost will most likely be decreasing in c.

[31] For expositional convenience, we ignored the set of parameters $c > \bar{c}$, the strategically rejected farmers. Eqs. (20) –(23) hold for values of c outside of this set. For values of c in this set, the density function remains $g(c)$ but those values of c produce no output. We account for this in the supply function below.

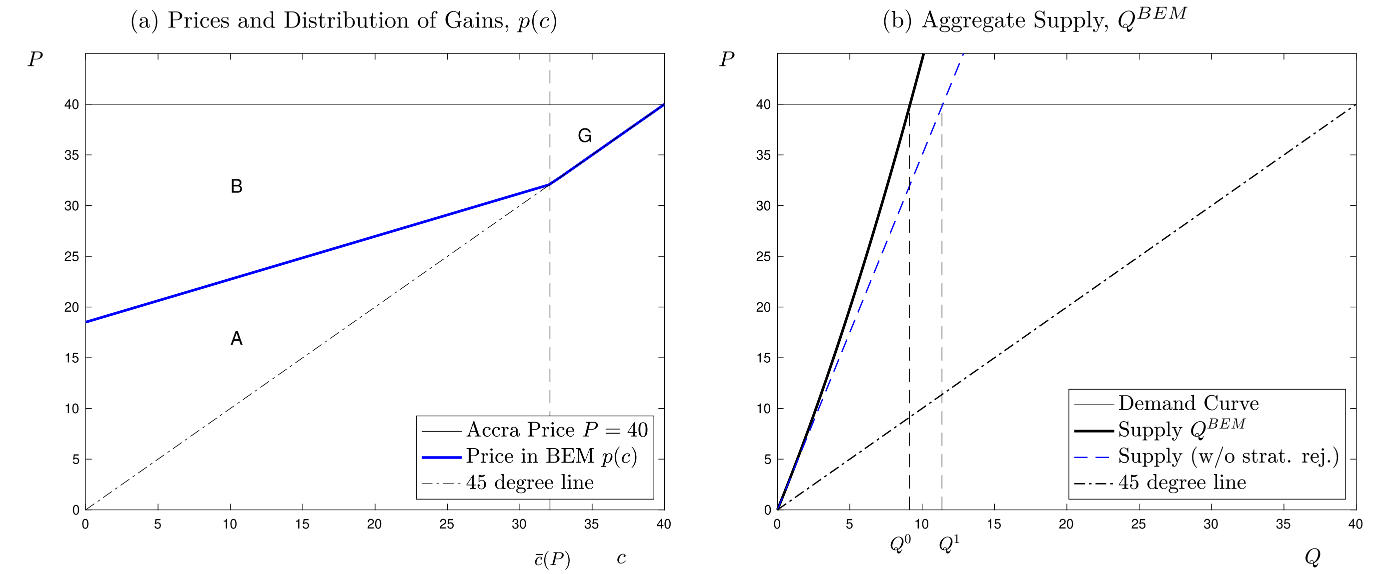

Fig. 4. Supply of agricultural goods in a bilateral exchange market. Notes: Panel (a) shows the distribution of prices obtained by farmers with different cost c. Panel (b) shows the aggregate supply of agricultural produce. In Panel (b), $Q^0 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve that excludes the quantity produced by strategically rejected farmers crosses the demand curve at $P = 40$ and $Q^1 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{P}{c^{max}}$ is the point where the aggregate supply that adds the strategically rejected farmers crosses the demand curve at that same retail price. The 45 degree line in Panel (b) represents the conventional supply curve. We constructed this figure using $[c^{max}, P, \beta, \delta, \kappa, \mu^F, \mu^T, \phi] = [40, 40, 0.5, 0.952, 0, 0.2, 0.8, 1]$.

produced in the economy. The trade costs of farmers, whether matched or not, are uniformly distributed on $c \in [0, c^{max}]$. The aggregate supply of the economy $Q^{BEM}(P)$ is therefore

$$Q^{BEM}(P) = \left(\int_0^{\bar{c}(P)} \frac{m^*}{c^{max}} dc \right) = \frac{\bar{c}(P)}{c^{max}} \frac{\mu^F}{\beta + \mu^F}. \tag{25}$$

Fig. 4(b) draws familiar supply and demand figures based on our model. The demand curve is represented by the horizontal black line.[32] The supply curve is given in Eq. (25) and is represented by the solid black curve.[33] In our model, the types of farmers that are matched and so send produce to the market are those in the set $[0, \bar{c}(P)]$. Hence, if we did not have to worry about the matches and random separations of traders and farmers, the total output at any price P would be $\bar{c}(P)$. This can be seen in Eq. (25) when $\mu^F / (\beta + \mu^F) = 1$. To highlight the role of strategic rejection, we also add to the figure the supply curve for the situation where trader reach an agreement negotiation with any farmer whose cost c is below P—that is, if there is no strategic rejection. In that case, all farmers in the set $[0, P]$ would generate a market surplus and send their produce to the market when they are matched to a trader. We also include one other potential supply curve, the conventional supply curve without any search and without the destruction of matches. This of course would be the 45 degree line, which is also drawn in the figure.

4. The dual market economy with the commodity exchange market (CEM)

This section introduces a commodity exchange market (CEM) into the economy. As we discussed earlier in Section 2.4, the commodity exchange has some brokers who receive produce from the farmers and other brokers who sell the produce to final buyers at a price P. Farmers can bring their produce and sell it to the members of the commodity exchange if they pay a transaction cost of τ. The commodity exchange has an auctioneer whose task is to equalize the demand from its members with the supply of farmers who brought their produce. This auctioneer operates as a Walrasian auctioneer. The price at the commodity exchange is the final price P, what we referred to earlier as the Accra price.

4.1. Value function of farmers in the CEM

Analogous to how we modeled unmatched farmers in the BEM, unmatched farmers who choose to sell to the commodity exchange have to wait a period, thereafter they can receive a value of $P(1-\tau) - c$ in each period.[34] The total discounted return of selling their produce to the commodity exchange is therefore:‘

$$V^{FCE}(c) = \frac{\delta}{1-\delta}(P(1-\tau) - c). \tag{26}$$

4.2. Definition of equilibrium in dual market

Definition 2. An equilibrium for this model with the given parameter set $\Omega = \{c^{max}, P, c, \beta, \delta, \kappa, \mu^F, \mu^T, \phi, \tau\}$ is a BEM equilibrium as in the earlier definition for a set of c's (which may be empty or the set of all c's) such that on the set where the BEM holds, conditions (1)–(5) with (26) replacing (4) hold and we add a new condition (6): Each farmer of cost parameter c decides whether to go to the CEM or BEM depending upon whether $V^{FCE}(c)$ is bigger than $V^{FU}(c)$ or not.

32 For expositional convenience we assume to be perfectly elastic, which has some justification as the country is small with imports of grains internationally and from the 3 countries it shares land borders with; it is of course easy to draw a downward sloping demand curve in its place.

33 Note that we set $\kappa = 0$ to ensure that there are always mass of farmers selling their produce to traders for any $P > 0$. With $\kappa > 0$, we have a discontinuity in the supply function at $c = \kappa$ since for low enough prices we have no trade between farmers and traders.

34 The timing per se is not too relevant, it would scale up or down the value functions by a fixed constant factor depending on the discount factor, but we do need to stick one timing scheme and keep it consistent between the BEM and CEM for the valid comparisons of the different regimes. (The scaling factor if all trades take place in the same period would be $\frac{\delta}{1-\delta}$, or its reciprocal.)

4.3. Proof of the existence of equilibrium in dual markets

To start, fix any value of V^{TU} in the BEM, and for any farmer with cost parameter c, perform all the substitutions in Eqs. (6)–(14) so that we have all our potential equilibrium variables $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$ stated in terms of V^{TU}. These satisfy all of the required Eqs. (1)–(5) except for Eq. (4). For any farmer type c, if that farmer goes into the BEM, those equations will hold for that c. This procedure replicates the proof for the existence of an equilibrium in the BEM model.

Different from our previous proof, here we need to determine, for each farmer of cost c, whether she goes to the CEM or the BEM. Specifically, the set of farmers who sell to the BEM will be a function of the equilibrium value of V^{TU}. Next, we derive the equations definition farmers' market choices. We then turn to the fixed point problem of V^{TU}.

Farmers' choice between the BEM and CEM and the partition of c. From Eq. (14), the value function $V^{FU}(c)$ of the farmer in the BEM is decreasing and linear in c (for $c < \bar{c}$) with slope given respectively by

$$V^{FU}_{slope} = -\frac{\delta\mu^F\phi}{(1-\delta)\sigma_1}.$$

For $c \geq \bar{c}$, $V^{FU}(c)$ is zero. Similarly, from Eq. (26), the value function $V^{FCE}(c)$ of the farmer in the CEM is decreasing and linear in c with slope given by

$$V^{FCE}_{slope} = \frac{\delta}{\delta - 1} \tag{27}$$

Claim 4. *The slope of $V^{FCE}(c)$ is steeper than that of $V^{FU}(c)$: i.e., $abs\left(V^{FCE}_{slope}\right) > abs\left(V^{FU}_{slope}\right)$.*

Proof. In the appendix. □

The claim above shows that, from a farmers' perspective, the value of being unmatched in the BEM does not fall as much with c as the value of selling to the CEM. This occurs because, as c rises, the present value of all the reductions in the future streams of sales must be discounted by the fact that farmers do not sell any produce in periods in which they are unmatched. This differential effect of c on $V^{TU}(c)$ and $V^{FCE}(c)$ gives rise to an intersection point which we define by $\underline{c}$. Simple algebra shows this to be:

$$\underline{c} = P\left(\frac{\tau_2 - \tau}{\sigma_1\sigma_2}\right) + \frac{\mu^F\phi(1-\delta)}{\sigma_2}V^{TU} \tag{28}$$

where

$$\tau_2 \equiv \frac{\sigma_2}{\sigma_1}. \tag{29}$$

When CEM and BEM coexist, $\underline{c}$ is the cutoff value below which farmers choose to sell to the CEM. As in the pure BEM case, we also have $\bar{c}$, given by Eq. (13), which is the cost above which farmers are strategically rejected by traders.

For the next claim, we define

$$V^{TU}_{\underline{c}} = -P\left(\frac{\tau_2 - \tau}{\mu^F\phi(1-\delta)\sigma_1}\right), \tag{30}$$

which is the value of V^{TU} when $\underline{c} = 0$ in expression (28). For any value of V^{TU} below $V^{TU}_{\underline{c}}$, no farmer chooses to sell to the CEM.

Claim 5. *$\bar{c}$ and $\underline{c}$ are functions of V^{TU} and have the following properties:*

1. $\bar{c}$ is linear and decreasing in V^{TU} with value P at $V^{TU} = 0$;
2. $\underline{c}$ is linear and increasing in V^{TU} with value $P(\frac{\tau_2-\tau}{\sigma_1\sigma_2})$ at $V^{TU} = 0$;
3. If $\tau < \tau_2$, then $\underline{c} > 0$ for all V^{TU} and, if $\tau > \tau_2$, then $\underline{c}$ is negative for V^{TU} in $[0, V^{TU}_{\underline{c}})$ and positive for V^{TU} in $(V^{TU}_{\underline{c}}, \infty)$;
4. $\underline{c}$ and $\bar{c}$, as functions of V^{TU}, have a common point of intersection at $V^{TU} = P\tau/(1-\delta)$, where they equal $P(1-\tau)$.

Proof. (1)–(4) follow from the definitions in (13) and (28)–(30). □

Fig. 5 illustrates the results from Claim 5. There are three partitions of c depending upon the values of $\bar{c}$ and $\underline{c}$ in relation to each other and to whether $\underline{c}$ is positive or negative. One can think of Fig. 5 as follows. Think of the $V^{FCE}(c)$ function (in yellow) as fixed. Then the $V^{FU}(c)$ function, which we know is flatter than the $V^{FCE}(c)$ function (in red), can be in one of three positions relative to the V^{FU} function: it can intersect it (Case I), it can lie above it (Case II) or lie below it (Case III). Each of these positions gives a different configuration of the set of farmers who choose to sell to traders in the BEM.

Case I $(0 \leq \underline{c} \leq \bar{c})$ (dual markets). In Case I, we have a set $[0, \underline{c})$ of c's where the CEM dominates the BEM and another interval $(\underline{c}, \bar{c})$ where the BEM dominates the CEM. Case I is a Dual Market economy. Here, the $V^{FU}(c)$ function intersects the $V^{FCE}(c)$ function, and this intersection occurs when either $V^{TU}_{\underline{c}}$ is negative and V^{TU} is in $(0, \frac{P\tau}{1-\delta})$ or $V^{TU}_{\underline{c}}$ is positive and V^{TU} is in $(V^{TU}_{\underline{c}}, \frac{P\tau}{1-\delta})$.

Case II $(\underline{c} < 0 \leq \bar{c})$ (pure bem). This is the case where the $V^{FU}(c)$ function lies above the $V^{FCE}(c)$ function, and occurs when $V^{TU}_{\underline{c}} > 0$ and V^{TU} is in the set $[0, V^{TU}_{\underline{c}})$. In Case II the BEM dominates the CEM for all relevant values of $c > 0$. Case II is a BEM only economy.

Case III $(\bar{c} < \underline{c})$ (pure CEM). This is the case where the $V^{FU}(c)$ function lies below the $V^{FCE}(c)$ function, and occurs when $V^{TU} > \frac{P\tau}{1-\delta}$. In Case III, the CEM dominates the BEM for all relevant values of $c > 0$. This case will not feature in the equilibria we will describe later. The reason is that in this case there would be no BEM and V^{TU} would not be well defined.

Defining and solving the fixed point problem for V^{TU}. To define the fixed point problem, as in the pure BEM case, we work with expression (4). The limits of integration differ depending upon whether we are in one of the three types of partitions of c as defined by Case I, II or III described above. We shall define first the integral if we were in Case I and define the expression as IV^{TM}_{DUAL}:

$$IV^{TM}_{DUAL} = \int_0^{\underline{c}} V^{TU}g(c)dc + \int_{\underline{c}}^{\bar{c}} \tilde{V}^{TM}(c)g(c)dc + \int_{\bar{c}}^{c^{max}} V^{TU}g(c)dc \tag{31}$$

where, recall, $g(c) = 1/c^{max}$, and

$$V^{TU}_{RHS,DUAL} = \delta(\mu^T IV^{TM}_{DUAL} + (1-\mu^T)V^{TU}) - \kappa. \tag{32}$$

Using the expression for $\tilde{V}^{TM}(c)$, it is easy to verify that $V^{TU}_{RHS,DUAL}$ is a quadratic function of V^{TU}. Let $a_{quad,DUAL}$ be the coefficient of the quadratic term of $V^{TU}_{RHS,DUAL}$ and $a_{const,DUAL}$ be the constant term of this quadratic function, simple algebra gives:

$$a_{quad,DUAL} = \frac{\delta\mu^T(1-\delta)^2\sigma_1}{2c^{max}\sigma_2^2}, \tag{33}$$

and

$$a_{const,DUAL} = \kappa^{Dual}_{max} - \kappa, \tag{34}$$

where

$$\kappa^{DUAL}_{max} = \frac{\delta P^2\mu^T\tau^2\sigma_1}{2c^{max}\sigma_2^2}. \tag{35}$$

Analogous to Claim 3, using Eqs. (33) and (34), we get the properties of $V^{TU}_{RHS,DUAL}$ listed in the next claim.

Claim 6. *The function $V^{TU}_{RHS,DUAL}$ has the following properties:*

1. $V^{TU}_{RHS,DUAL}$ is convex in V^{TU};

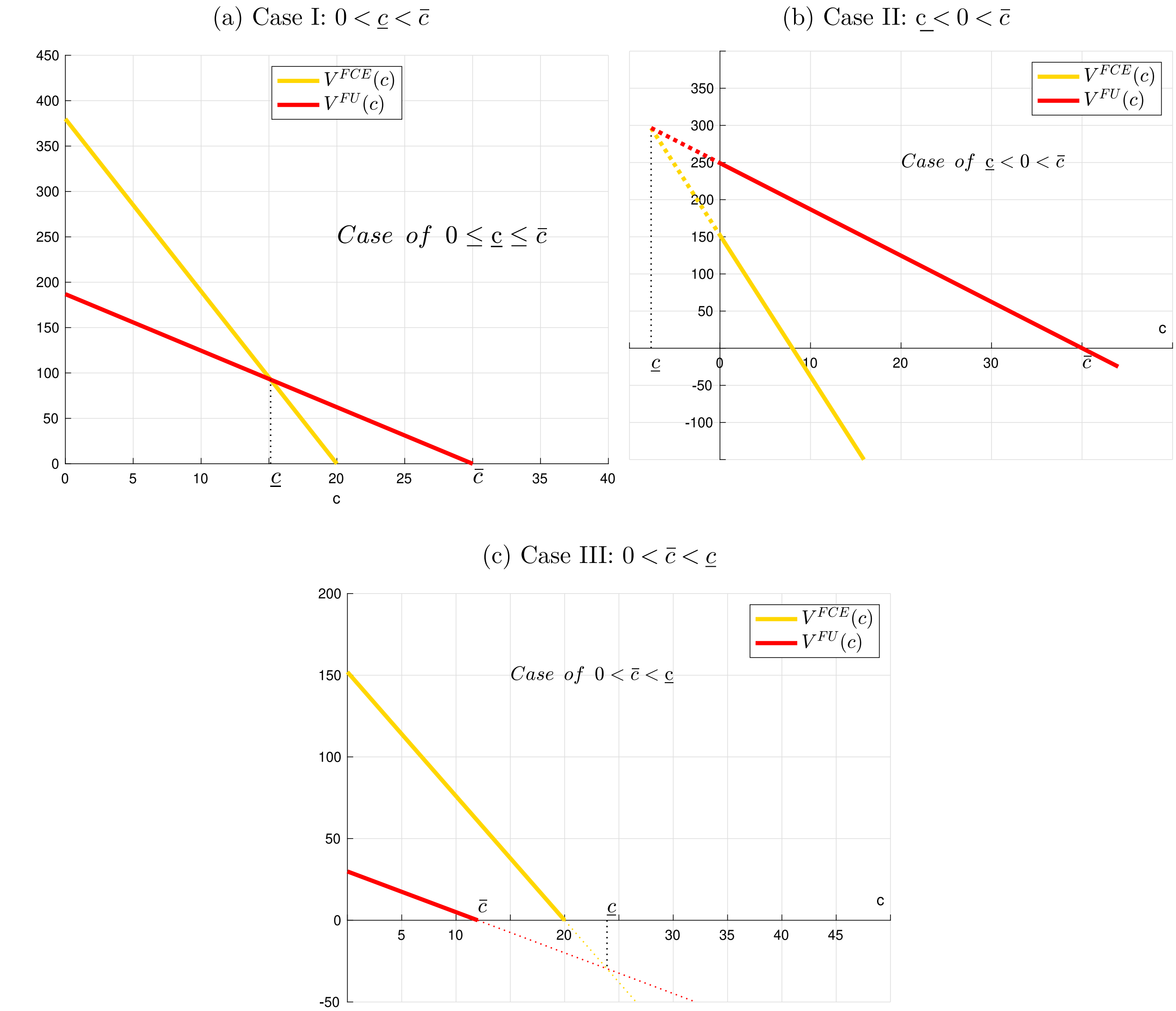

Fig. 5. Relationship between $\bar{c}$ and $\underline{c}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2. if $\kappa < \kappa_{max}^{DUAL}$, then $V^{TU} - V_{RHS,DUAL}^{TU} < 0$ at $V^{TU} = 0$, and, if $\kappa > \kappa_{max}^{DUAL}$, then $V^{TU} - V_{RHS,DUAL}^{TU} > 0$ at $V^{TU} = 0$;
3. at $V^{TU} = \frac{P\tau}{1-\delta}$, $V^{TU} - V_{RHS,DUAL}^{TU} = P\tau + \kappa > 0$.
4. As a function of V^{TU}, the function $V_{RHS,DUAL}^{TU}$ has a unique fixed point on $[0, \frac{P\tau}{1-\delta})$.

Expression (32) would be the correct variant of the right-hand side of (4) if Case I holds. If Case II holds, since the BEM is chosen for all c, then expression (16) would be the correct variant of the right-hand side of (4). When Case III holds, the CEM is chosen at all values of c.

Different from our proof strategy in the case of a pure BEM, Eq. (4) now depends on the configuration of farmers who select into each type of market. For clarity, let us define $V_{RHS,BEM}^{TU}$ as the variant of V_{RHS}^{TU} which occurs when all farmers go to the BEM. By definition, at $V^{TU} = V_{\underline{c}}^{TU}$, we have $\underline{c} = 0$ and, at this value, $V_{RHS,DUAL}^{TU} = V_{RHS,BEM}^{TU}$. Based on this, we define $\overline{V}_{\underline{c}}^{TU} \equiv V_{RHS,DUAL}^{TU} = V_{RHS,BEM}^{TU}$ when $V^{TU} = V_{\underline{c}}^{TU}$. We now have the required elements for our next proposition.

Proposition 2. *Suppose that $\kappa < \min\{\kappa_{max}, \kappa_{max}^{DUAL}\}$. (I) Suppose that either $V_{\underline{c}}^{TU} > 0$ and $\overline{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$ or $V_{\underline{c}}^{TU} < 0$. Then there is a unique equilibrium and that equilibrium is a dual economy. (II) Suppose that $V_{\underline{c}}^{TU} > 0$ and $\overline{V}_{\underline{c}}^{TU} < V_{\underline{c}}^{TU}$. Then there is a unique equilibrium and that equilibrium is a BEM only economy.*

Proof. Fig. 6 provides intuition to our proof (see Appendix Section C for details). In each panel, we have two lines: a yellow one that represents $V_{RHS,DUAL}^{TU}$ and a red one that represents $V_{RHS,BEM}^{TU}$. Each of these two lines represent a different variant of the right-hand side (RHS) of Eq. (4). The point where they intersect each other gives $V_{\underline{c}}^{TU}$ in the horizontal axis and $\overline{V}_{\underline{c}}^{TU}$ in the vertical one. Fig. 6(a) illustrates the case in which the two lines intersect each other at a point that is above the 45 degree line. In this case, we know that $\overline{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$, which implies that the correct variant of the RHS of Eq. (4) is the yellow line, since some farmers choose to sell to the CEM and some to the BEM. Therefore, in Fig. 6(a) the solution to the of the system is given by V_{DUAL}^{TU*} instead of V_{BEM}^{TU*}. A similar logic can be applied to Fig. 6(b), in which the two lines intersect each other below the 45 degree line. In this second case, the correct variant of the RHS of Eq. (4) is $V_{RHS,BEM}^{TU}$ and we can rule out V_{DUAL}^{TU*} as a solution to the problem. Once we have identified the correct variant of the RHS of Eq. (4), we can then follow steps very similar to that used in proving the existence of equilibrium in the pure BEM economy in Proposition 1. We therefore obtain the existence of an equilibrium in the economy

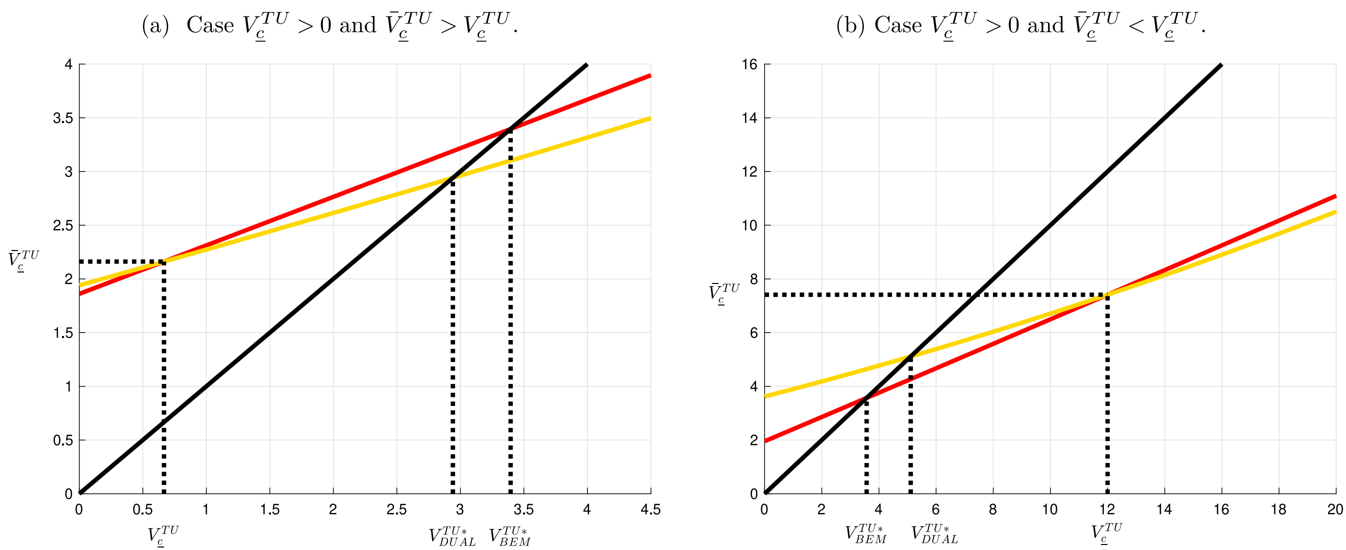

Fig. 6. Different cases for V_{BEM}^{TU*} and V_{Dual}^{TU*}. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where both the CEM and BEM are possible. The conditions (I) and (II) of this proposition then gives us the situations where we have a dual or a BEM only economy. □

Proposition 2 shows the existence of an equilibrium value of V^{TU}. This value of V^{TU} provides us with equilibrium values of all the other value functions as well as the pricing function $p(c)$, by retracing the steps we took to get V^{TU}. Fig. 6(a), which was drawn with parameter values $[c^{max}, P, \beta, \delta, \kappa, \mu^F, \mu^T, \phi, \tau] = [40, 40, 0.1, 0.5, 0.005, 0.9, 0.5, 2, 0.4]$, shows the equilibrium value of V^{TU} to be $V^{TU*} = 2.94$. Retracing our steps, we get: $\underline{c}^* = 2.73$, $\bar{c}^* = 38.53$, $V^{FU*} = 27.20 - 0.71c$, $\tilde{V}^{TM*} = 18.06 - 0.39c$, $p(c) = 30.21 + 0.22c$, and $V^{FM*} = 57.42 - 1.49c$. Fig. 6(b), drawn with parameter values $[c^{max}, P, \beta, \delta, \kappa, \mu^F, \mu^T, \phi, \tau] = [40, 40, 0.1, 0.5, 0.1, 0.9, 0.5, 2, 0.3]$, shows the equilibrium value of V^{TU} to be $V^{TU*} = 3.57$. Retracing our steps, we get: $\underline{c}^* = -10.12$, $\bar{c}^* = 38.22$, $V^{FU*} = 26.98 - 0.71c$, $\tilde{V}^{TM*} = 18.56 - 0.39c$, $p(c) = 29.97 + 0.22c$ and $V^{FM*} = 56.95 - 1.49c$.

4.4. Aggregate supply and gains from trade in dual market economy

We now describe the characteristics of the equilibrium in the economy with the potential for dual markets. We will look at the distributional gains from trade just as we did earlier in the economy where only the BEM was allowed.

Gains from trade in dual markets. Fig. 7(a) shows the price function in the Dual Market Economy. The figure highlights the gains from trade between farmers, traders and the commodity exchange.[35] Area B represents the revenues generated by the fee of the CEM, area A represents the profits of farmers who sell to the commodity exchange, area D the surplus of traders in the BEM and area E the profits of farmers who sell to the BEM. Area G captures the loss of potential matches due to strategic rejection.

At $\underline{c}$, there is a gap between the price obtained by farmers in the CEM by farmers and the price obtained by farmers in the BEM. This gap captures the tradeoff between the certain price obtained by farmers in the CEM and the price volatility in the BEM. When farmers negotiate with traders, they incorporate in their utility value the probability of not being matched in future periods, i.e., the probability of incurring post-harvest losses. This occurs because there exists an inherent risk in BEM that is internalized in the negotiated price between traders and farmers. Later, in Section 4.7, we discuss in more detail how risk-aversion shapes this price gap.

Aggregate supply in dual markets. In the dual economy we have farmers with cost in $[0, \underline{c})$ selling to the CEM and those with cost in $(\underline{c}, \bar{c})$ selling to the BEM. All farmers with cost in $[0, \underline{c})$ are "matched" (to the CEM) with probability one. For farmers who sell to the BEM, we have a steady state probability m^* of being matched which is the same as the one that we derived in the pure BEM market (see Eq. (23)). As in Section 3.4, we assume that the set of all farmers—matched and unmatched—is uniform on $[0, c^{max}]$ and that there is a unit mass of farmers. The aggregate supply in the dual economy, $Q^{DUAL}(P)$, is thus given by[36]

$$Q^{DUAL}(P) = \int_0^{\underline{c}(P)} \left(\frac{1}{c^{max}}\right) dc + \int_{\underline{c}(P)}^{\bar{c}(P)} \left(\frac{m^*}{c^{max}}\right) dc = \frac{\underline{c}(P)}{c^{max}} \frac{\beta}{\beta + \mu^F} + \frac{\bar{c}(P)}{c^{max}} \frac{\mu^F}{\beta + \mu^F} \quad (36)$$

Fig. 7(b) illustrates the aggregate supply using the same parameter values as in Fig. 4. The thick black line shows the supply curve, $Q^{DUAL}(P)$, in the dual market economy. We have also shown, for the same parameter values, the supply curve in the economy where there is only one BEM market and the CEM is not allowed—that is the thin black line and is the same supply curve as in the BEM only economy of Fig. 4(b).

In dashed blue line we have the supply in the dual market economy all farmers below $\bar{c}$ sold their produce to the BEM and incurred the post harvest losses associated with not being matched with traders. In this case, only a share $\mu^F / (\beta + \mu^F)$ of all farmers would sell to BEM markets in steady state. The remainder are those who are unmatched. With the commodity exchange, farmers with costs below $\underline{c}$ sell to the CEM and are matched with probability one. The difference between the blue line and the thin black line is therefore the improvement in supply due to the role of the commodity exchange in changing the share of farmers with a match from the fraction $\mu^F / (\beta + \mu^F)$ to the fraction 1. (One may ask why the dotted blue line is not the same as the thin black line. After all, they are both situations where only the BEM operates. The reason is that they are two different equilibria with different values of $\bar{c}$.)

[35] This graph is generated from the following parameters, chosen to make the graph and the different parts clear: $[c^{max}, \beta, \delta, k, \mu^F, \mu^T, \phi, \tau] = [40, 0.5, 0.952, 0, 0.2, 0.8, 1, 0.5]$.

[36] This is for the case when $\underline{c} > 0$; when $\underline{c} \leq 0$ we replace $\underline{c}$ in Eq. (28) with 0, in which case the formula collapses to the expression for the BEM only economy discussed earlier (see (25)).

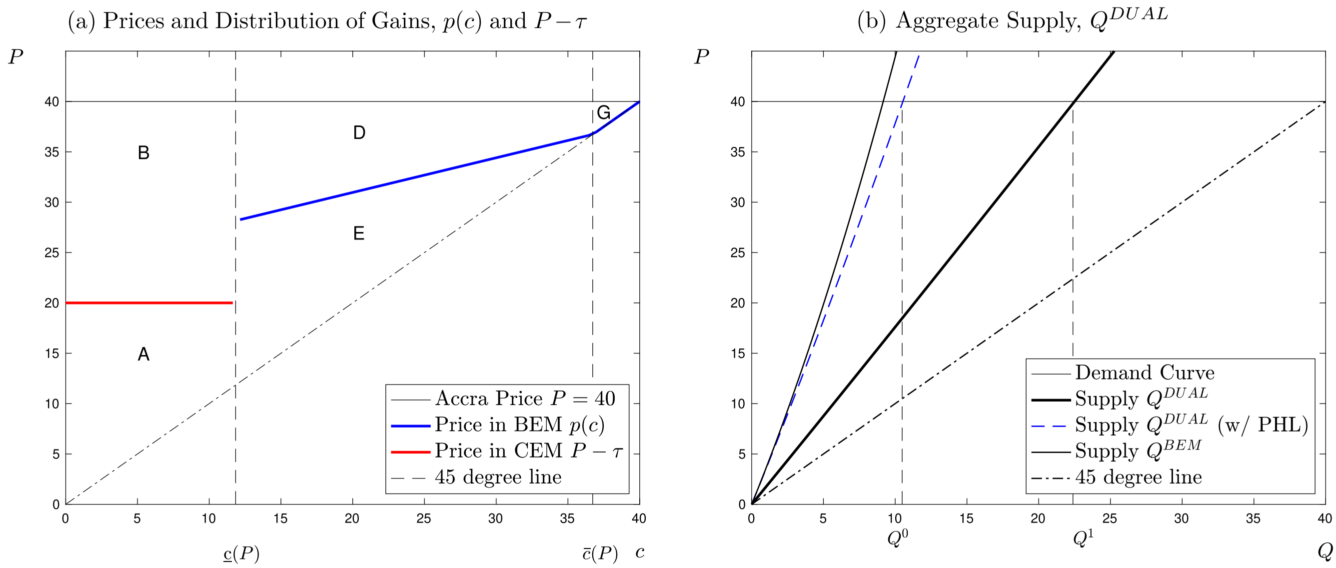

Fig. 7. Aggregate supply of agricultural goods in a dual market. Notes: Panel (a) shows the distribution of prices obtained by farmers with different cost parameter c. Panel (b) shows the aggregate supply of agricultural produce. In the figure, $Q^1 \equiv \frac{\beta}{\beta+\mu^F}\frac{\underline{c}}{c^{max}} + \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve in dual markets crosses the demand curve at $P = 40$ and $Q^0 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve in dual markets, excluding the post-harvest losses (PHL) avoided by farmers when they sell to the CEM, crosses the demand curve at that same price.

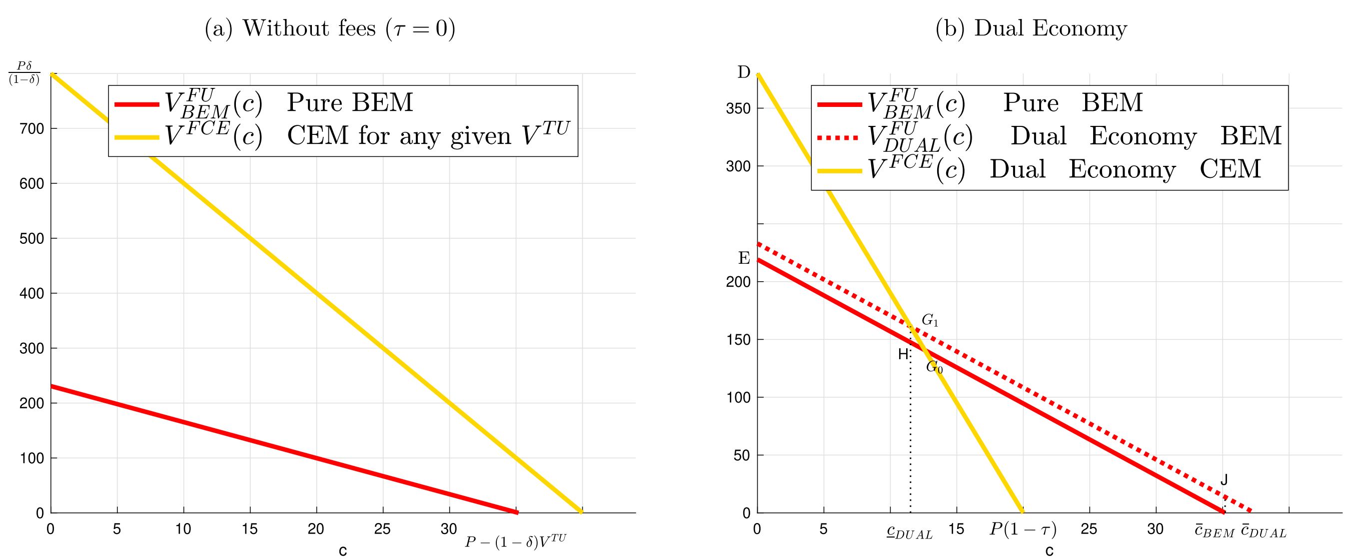

Fig. 8. Gains from introducing a CEM. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4.5. Welfare gains from the introduction of the commodity exchange market

Let us now consider the welfare gains from introducing the CEM in the economy. To start, set $\tau = 0$. Fig. 8(a) shows the welfare of unmatched farmers with different cost c when $\tau = 0$. In that case, it is easy to see that the commodity exchange is unambiguously better than the BEM for all farmers.

With a positive fee and a dual economy, comparing the welfare requires some caution since the introduction of the CEM changes the operation of the BEM in the dual economy. In other words, the BEM itself will change when it goes from being the only market to being one of two markets together with the CEM.

Fig. 8(b) draws in yellow the value function of the farmer in the CEM, $V^{FCE}(c)$, given by Eq. (26). It also draws the value function of the farmer in the BEM, $V^{FU}(c)$, given by Eq. (14). There are two variants of this value function. We draw the value function in equilibrium in the scenario in which there is only a BEM, which we define as $V^{FU}_{BEM}(c)$. This value function gives some equilibrium value of V^{TU} equal to V^{TU*}_{BEM}. With the introduction of the CEM, there is a change in the equilibrium and a new value of V^{TU}, which we define as V^{TU*}_{DUAL}. We denote by $V^{FU}_{DUAL}(c)$ the value function of farmers in that case. As shown earlier in Fig. 6(a), with dual markets, $V^{TU*}_{DUAL} < V^{TU*}_{BEM}$. An immediate inspection of $V^{FU}(c)$ in Eq. (14) shows that, for $V^{TU*}_{DUAL} < V^{TU*}_{BEM}$, $V^{FU}_{DUAL}(c)$ is above $V^{FU}_{BEM}(c)$ for any c.

Fig. 8(b) shows that the pure BEM economy has a lower value for unmatched farmers (the thick red line) than the BEM in a dual economy (the dotted red line). The reason for this is that the equilibrium value for unmatched traders, V^{TU*}, is higher in the pure BEM model in comparison with the dual economy. Unmatched traders do better in the pure BEM economy than in the dual market economy. This is because low-cost farmers prefer to sell to the CEM, and such farmers are the ones with whom traders obtain the largest gains from trade in the pure BEM. Since these farmers are no longer available to traders, they need to settle for lower utility values in the dual economy. As such, they also lower their standards for strategic rejection, and start accepting farmers with higher costs c, which is shown in the figure by a move from $\bar{c}_{BEM}$ to $\bar{c}_{DUAL}$. We summarize these results in the next proposition.

Proposition 3. *Assume that the conditions of Proposition 2(I) hold, so that we have the existence of a CEM in the dual economy. As defined earlier, let V^{TU*}_{BEM}, $V^{FU}_{BEM}(c)$ and $\bar{c}_{BEM}$ be the value function of the unmatched traders, the value function of the unmatched farmers and the cutoff strategic rejection point in the economy prior to the introduction of the commodity exchange, and let the equivalent values after the introduction of the commodity exchange be V^{TU*}_{DUAL}, $V^{FU}_{DUAL}(c)$ and $\bar{c}_{DUAL}$. Then (a) $V^{TU*}_{BEM} > V^{TU*}_{DUAL}$; (b) , $V^{FU}_{BEM}(c) < V^{FU}_{DUAL}(c)$ for all c; and (c) $\bar{c}_{BEM} < \bar{c}_{DUAL}$.*

The different areas in Fig. 8(b) underscore the gains for different sets of farmers. It shows that farmers in the set $[0, \underline{c}_{DUAL}]$ move from the BEM to the CEM. Their benefit in terms of their value functions is the area $HEDG_1$. Those with costs in the set $[\underline{c}_{DUAL}, \bar{c}_{BEM}]$ stayed in the BEM sector in both the pure BEM and the dual economy models. However, the BEM is better for the farmers in the dual economy, so they too obtain a benefit represented by the set $HG_1J\bar{c}_{BEM}$. Finally, there is the set of farmers with costs in $[\bar{c}_{BEM}, \bar{c}_{DUAL}]$ who were not served in the original pure BEM economy but are now served in the BEM sector of the dual economy. The benefit of those farmers is represented by the area in the triangle formed $\bar{c}_{BEM}J\bar{c}_{DUAL}$. There is a set of farmers in the set $[\bar{c}_{DUAL}, P]$ with no change in welfare, since they remain without options to sell their produce.

We now turn to our second measure of welfare: the aggregate mass of matches in the economy in every period. This measure of welfare, of course, ignores the costs of traders and the commodity exchange—which we discuss next. Earlier we indicated the aggregate supply in the pure BEM economy, Q^{BEM}, in Eq. (25) and that for the dual economy, Q^{DUAL}, in Eq. (36). It is easy to see that

$$Q^{DUAL} - Q^{BEM} = \left(\frac{1}{c^{max}}\right)\left\{\underbrace{\int_0^{\underline{c}_{DUAL}} dc + m^* \int_{\underline{c}_{DUAL}}^{\bar{c}_{DUAL}} dc}_{\text{Matches in Dual}} - \underbrace{m^* \int_0^{\bar{c}_{BEM}} dc}_{\text{Matches in BEM}}\right\}.$$

After simple rearrangements, the equation above becomes

$$Q^{DUAL} - Q^{BEM} = \left(\frac{1}{c^{max}}\right)\left\{\underbrace{\int_0^{\underline{c}_{DUAL}} (1-m^*)dc}_{\text{Direct mechanism}} + \underbrace{\int_{\bar{c}_{BEM}}^{\bar{c}_{DUAL}} m^* dc}_{\text{Indirect mechanism}}\right\} \quad (37)$$

The first term inside brackets captures what we refer to as the "direct" impact of the CEM. It represents the additional matches for farmers with cost in $[0, \underline{c}_{DUAL}]$ who move from the BEM to the CEM. A mass equal to $1 - m^*$ of such farmers would be incurring post-harvest losses in the absence of the CEM. Eq. (37) also highlights a second welfare channel, which we refer to as the "indirect" impact of the CEM. There is a set farmers with cost parameters in the set $[\bar{c}_{BEM}, \bar{c}_{DUAL}]$ who were not served by the BEM in the pure BEM economy, but that are now served in the dual economy. The probability of match, and hence expected output of each of such farmers is m^*.

Eq. (37) provides a transparent measure of the new matches in the economy, but it does not incorporate the costs of the CEM (τ) or the trade costs of farmers (c). The next expression gives the total gains from CEM, defined by ΔW

$$\Delta W = \left(\frac{1}{c^{max}}\right)\left\{\int_0^{\underline{c}_{DUAL}} (P-c)(1-m^*)dc + \int_{\bar{c}_{BEM}}^{\bar{c}_{DUAL}} (P-c)m^* dc - \int_0^{\underline{c}_{DUAL}} \tau dc\right\}.$$

This expression gives the flow of aggregate production value in the economy. To obtain the infinite discounted value of such flows, we have to compute their present value by dividing terms by $(1 - \delta)$.[37] Here, the interpretation of τ matters for our discussion. If we interpret τ as a transfer from farmers to the CEM—as we treat the difference between P and $p(c)$ for the transactions between farmers and traders—, then the CEM will unambiguously increase the total value produced in the economy. However, if τ is treated as production cost, then the impact of the CEM on the total value of production is ambiguous.[38]

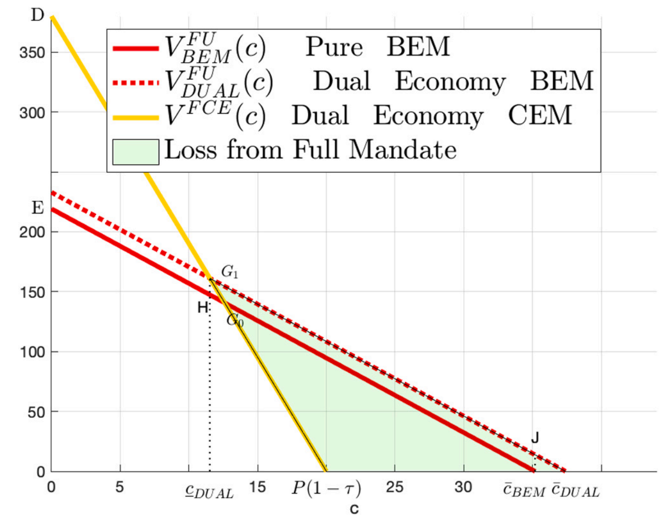

Fig. 9. Full versus partial mandate.

4.6. Full or complete mandates and the resilience of the traditional bilateral exchange market

Related to our main result, Proposition 2, are two subtle observations, one with important policy implications. The first observation has to do with our result and definition of an equilibrium. Our proposition establishes that, depending upon parameter values, there exists either a pure BEM or else a dual economy. Suppose now that the government outlaws the BEM. This is what the government of Ethiopia has done for several key crops in the economy. If the BEM is banned, then of course the CEM will exist, but for a different set of farmers.

Indeed consider Fig. 9. We draw the value functions of the farmers in the dual economy equilibrium. We could ask what would happen if instead of allowing a partial mandate (case of Ghana and Malawi) instead one would legislate a complete full mandate (case of Ethiopia). In the partial mandate the farmers served would be those of cost from 0 to $\underline{c}_{DUAL}$ indicated in the figure. In the full mandate system, the CEM will be the only one which will operate, so the set of farmers would be $[0, P(1-\tau))$, which is larger than the set of farmers in the CEM in the partial mandate system. However, there will be a set of farmers, those in the set $(P(1-\tau), \bar{c}_{DUAL})$, that would have been served by the BEM in a dual market but are not being allowed to trade. Farmers with cost in the set $(\underline{c}_{DUAL}, \bar{c}_{DUAL})$ would have gone to the BEM and obtained higher payoffs than in the full mandate model. All high-cost

[37] Since utility functions are linear in the value of goods, the welfare gain measured according to the aggregate value produced in this economy is directly related to the utilitarian welfare.

[38] In addition, to simplify the discussion, we assumed in the expression above that $\kappa = 0$. If $\kappa > 0$, we would have to take into account that in the CEM there is a larger mass of traders who spend κ in every period to search for farmers but who are not matched to any farmer. Specifically, we would have to add two additional terms: (i) one term related to the search costs incurred by traders who are no longer matched to low-cost farmers who sell to the CEM (i.e., $-\kappa\underline{c}_{DUAL}$), and (ii) another term related to the additional farmers who are matched to traders when there is a CEM (i.e., $(\bar{c}_{DUAL} - \bar{c}_{BEM})\kappa$).

farmers in the set $[P(1-\tau), P]$ would still be left behind and out of the CEM. The area shaded green is therefore the loss to farmers when the dual economy model is forced to be a CEM only model due to the full mandate legislation.

In Ethiopia, for the crops which have access to the commodity exchange and are subject to the full mandate, many have observed the existence of a "black market" where some farmers continue to illegally sell their produce through intermediaries, which is the equivalent of the BEM modeled here.[39] The result we have just described explains why such a black market would exist. Our model also predicts that it is the high cost farmers who would be engaged in the black market. We underscore that, despite this legal requirement, the existence of dual markets is evidence of the desire of farmers to be in a BEM. If the laws were removed or relaxed we would see an even larger BEM than we actually do see in Ethiopia.

The second observation is that our existence result shows that when the BEM is allowed, then regardless of how small the inefficiency of the CEM is, there will always be a BEM in operation. One would conjecture that the smaller is the inefficiency (the fee τ) the smaller is the size of the BEM within the dual economy.

For the existence of the BEM, we know from our earlier propositions that the entry cost κ must be small. We will require the cost to be less than κ_{max}^{BEM} in Eq. (19) and also less than κ_{max}^{DUAL} in Eq. (35). The first bound, κ_{max}^{BEM}, is independent of τ. The second is a function of τ and, for a given κ, we require τ to be sufficiently large for the BEM to exist, and indeed this bound is given by Eq. (35) and equals

$$\hat{\tau} = \left(\frac{2c^{max}\sigma_2^2}{\delta P^2 \mu^T \tau^2 \sigma_1}\right)^{\frac{1}{2}} \kappa \tag{38}$$

An immediate corollary of our main Propositions is the following, which shows that, as long as the search cost κ is sufficiently small, there are always economic incentives for the existence of a BEM.

Corollary 1. *(a) Suppose $\kappa = 0$. Then, regardless of how small the value of τ is, there will always be a BEM sector in the economy. (b) If $\kappa > 0$, then there will be a BEM sector in the economy so long as the commodity exchange fee τ exceeds the value $\hat{\tau}$ in Eq.* (38).

We conclude this section by emphasizing that three ingredients are critical for the analytical results in this paper: (i) the search costs related to the probability of not forming a match, (ii) the heterogeneous cost parameters of farmers, and (iii) the commodity exchange market fees. The probability of not forming a match in the BEM generates incentives for some farmers to sell to the CEM. Heterogeneous costs, interacted with the CEM fees, determine which farmers sell to the BEM and which sell to the CEM. Appendix Section E provides comparative statistics for different parameter values and show how the main results of the paper change when we change the parameters related to these three ingredients. In particular, we vary the values of c^{max}, τ, μ^F, and μ^T, and inspect the effects on $\bar{c}$, $\underline{c}$ and the gains from the CEM, as captured by Eq. (37).

4.7. Implications of risk-aversion

We now turn to discuss the economic implications of introducing risk aversion on the part of farmers. First, notice that, when farmers have linear utilities, they do not value consumption smoothing and their choices are entirely based on the present value of their future streams of income. In that scenario, the reason why some farmers choose to sell to the CEM is because the lack of post-harvest losses increases the present value of their income, and not because of the volatility that the post-harvest losses bring to their consumption. When farmers have concave utilities, they have a preference for consumption smoothing. In that case, farmers value the CEM in part because the CEM allows them to reduce the volatility in their income. In sum, with concave utilities, the CEM brings two potential sources of benefits for farmers: (1) it can increase the present value of farmers' stream of income; (2) it can reduce the risk of post-harvest losses and allow them to smooth consumption.

To illustrate how concave utilities shape our results, we compare the market equilibrium in the presence of the CEM when farmers have linear utilities with the market equilibrium when farmers have concave utilities, using the constant relative risk aversion (CRRA) utilities.[40] Panel (a) of Fig. 10 plots the distribution of prices when utilities are linear and Panel (b) when utilities are concave, holding all other parameters constant. Our qualitative patterns remain the same. As in the linear case, low-cost farmers select into the CEM when they have concave utilities. As such, high-cost farmers are the ones left behind by the CEM and only indirectly benefited by it due to the changes in the BEM. Quantitatively, Fig. 10 highlights two key patterns. First, there is a larger fraction of farmers who would choose to sell to the CEM if utilities are concave. Second, for the farmer at $\underline{c}$ who is indifferent between the BEM and the CEM, there is a larger price gap between the BEM and the CEM. Both of these patterns are consistent with the fact that the CEM allows farmers to smooth consumption.

5. Robustness section

We employed different simplifying assumptions to focus on the inefficiencies generated by search costs. This section further discuss some of these assumptions, how they affect our results, and their empirical motivation. We cover four aspects of our model. First, the fact that traders match to at most one farmer. Second, the observability of the cost parameter c and its relationship to the search process of traders. Third, the different sources of price volatility implicit in our model. Lastly, the endogenous entry of traders.

One farmer to one trader match. In our model, one farmer matches to one trader. This formulation generates an important tradeoff: Traders compare the benefits of being immediately matched to a farmer against the expected value of being matched with farmers in the future. As a consequence, some high-cost farmers who would generate a positive market surplus (i.e., farmers whose cost satisfy $p > c$) are strategically rejected by traders. The results of our model are the same to the results of an alternative model in which traders match to a finite, but multiple number of farmers. If this were say, n farmers per trader, we could think of considering the one trader with n farmers as being technically n traders, each having one trader to deal with.[41] This becomes equivalent to our one to one model.

[39] We spoke to a number of high ranking current and former officials at the Ethiopian Commodity Exchange. Some typical responses are the following: "A the Mercato (an open air market, perhaps Africa's biggest) .. people are opening selling grade 1 and grade 2 in the open market. They see the police and they run away. That black market was created. After a while they let this alone—i.e., the government. The government put controls on the (coffee) washing centers and aggregation areas. You arrest the woman, the next day the husband shows up. You arrest the husband and the son shows up. This is because it is reserved for export. The middle class wants the good coffee".

[40] We emphasize that this exercise should be interpreted with care, since we do not prove formally how the endogenous variables of the model change with different parametrizations of CRRA.

[41] We highlight one technical issue here. We follow the vast majority of the literature in search and labor markets, which makes the assumption that firms operate a constant returns to scale technology. Specifically, we assume that the technology that traders use to convert agricultural goods purchased from farmers into goods sold at retail markets is constant returns to scale. This assumption makes the negotiation of a trader with a farmer independent from her negotiation with another farmer. As discussed in detail in Elsby and Michaels (2013), with decreasing returns to scale the marginal worker generates less surplus than infra-marginal ones. Therefore, the rent that a firm

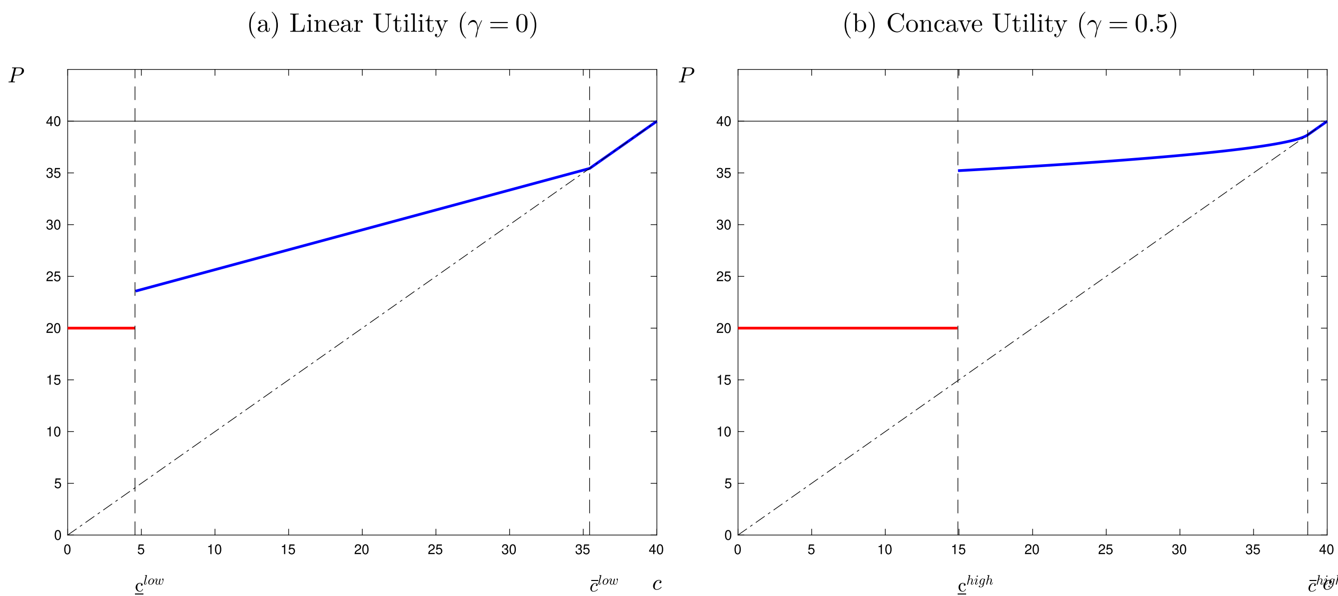

Fig. 10. Prices received by farmers under linear and concave utilities. Notes: Panel (a) plots equation $V^{TM}(c)$ when we have linear utilities and Panel (b) when we have concave utilities based on the CRRA utility with parameter γ. The rest of parameters are set at $[c^{max}, P, \beta, \delta, \mu^F, \mu^T, \phi, \kappa, \tau] = [40, 40, 0.2, 0.5, 0.75, 0.5, 0.5, 1, 0, 0.5]$. This figures shows how the equilibrium prices change when we change the curvature of the utility function. With larger γ, farmers value the commodity exchange more. In doing so, traders also extract lower value from farmers, which reduces the range of farmers who are strategically rejected by traders. See Panel (a) in Fig. 7(a) for additional description of the lines.

If we assumed the extreme case in which traders can be matched to an unlimited number of farmers, the tradeoff mentioned above would disappear. In that case, traders would always be able to generate new offers to farmers in the future, without losing the opportunity of trading with the ones they have been currently matched with, and there would be no strategic rejection. We believe that this extreme case, however, is inconsistent with our institutional setting. As described earlier in Section 2, traders in the agricultural markets of sub-Saharan Africa have limited capital and tend to be very small. They would not be able to handle the cash payments and the risk of larger volumes. Many traders either come into towns with their own small lorry, or else often carry the goods in a local private bus ("tro tro" or "Matatu"). That is, oftentimes, traders are even constrained by what they can physically carry themselves.

Search and observability of c. Traders often learn about trading opportunities by calling their relatives, friends and other traders. They also sometimes learn about these opportunities by asking around in the village during market days or by driving around trying to find farmers on the roadside of main roads. This search process comes with substantial uncertainty about the characteristics of farmers. As they search, traders have limited knowledge about the cost of the farmer that they might be able to find. Some of those farmers may need to bring their goods to the farm gate after hauling it over big hills, perhaps also a river or swamp. To capture this uncertainty in a parsimonious way, traders in our model imperfectly observe the farm specific cost c: They know the distribution of c in the village in which they are searching for farmers, but, *ex-ante*, they do not observe the specific value of c of the farmer that they might find.

In principle, one could extend our model to allow for traders who observe more information about the cost parameter c *ex-ante*. For example, suppose that there are two areas, call them A and B, with different distributions of the cost parameters, c. Area A could have primarily low c farmer types, while the other has high c types. Those with low c's will have higher potential gains to traders. If traders can choose which area they visit, they have to be indifferent between going to the two areas A and B, and the expected returns need to be the same. This would then mean that the places with higher potential gains should have lower probabilities of a match for the trader. Low c's would then imply lower probability of finding a match. (This is similar in spirit to the posted wages models—see, e.g., Rogerson et al. (2005)—in the sense that the expected returns to traders should be equated across different areas.)[42]

Aggregate vs farm-level price volatility. Conceptually, we think of two sources of price volatility in our model: (i) one source related to the aggregate or final price of produce in retail markets P; (ii) another source related to the price that farmers receive for their produce, which can be $p(c)$ or 0 depending on whether or not farmers sell their goods to traders. We modeled the second source and not the first. Of course, we could have gone further and attempted to model randomness in the final or "Accra" price P itself. One could ask: if we had gone the route of modeling the uncertainty in the aggregate or macro price P, what would happen? In that case, we would have to be specific about the timing and who has what price information at the time of the match and the bargaining. If they (the trader and the farmer) both have no information on P before trading, then they would replace the price P with the expected price $E(P)$, and the analysis would follow very similarly to what we have modeled in the case of risk-neutral farmers. However, if one side has better information than the other then we obtain a much more complicated search problem where the uninformed needs to infer the information of the informed.[43] Here, we abstracted from aggregate price volatility as to focus on the impact that

obtains with a marginal worker is no longer independent from the rents that the firm obtain with others, which requires different solution methods.

[42] In this less parsimonious version of the model, we would have to characterize several additional features in the model to prove existence and uniqueness of equilibria. For example, we would have to model how traders choose between sub-villages, and those traders would have to be indifferent between going to sub-villages where farmers have a low average c versus a village where farmers have a high average c. In equilibrium, the expected return from searching in these different sub-villages would have to equalize.

[43] See, e.g., Hildebrandt et al. (2015) which models this situation—in that paper the trader has better information than farmers (at least most of them).

a commodity exchange has on the ability of farmers to find traders (the question of post-harvest losses).[44]

Endogenous entry of traders. As discussed in Section 2, there are several factors limiting the entry of traders in agricultural markets in sub-Saharan Africa, among them, the existence of market queens or capital constraints. Based on this observation, we assumed a fixed mass of traders in our model, which contrasts with the common assumption of free entry of firms adopted in search models of labor markets. What would happen to the existence of strategic rejection and the dual markets if there were free entry of traders? If we assumed free entry of traders with a zero profit condition, that would drive V^{TU} to zero, since a positive V^{TU} would lead to an inflow of traders to the region. Because $\bar{c} = P - (1-\delta)V^{TU}$, the free entry of traders would imply $\bar{c} = P$ and remove any strategic rejection. We think that this extreme assumption, however, is inconsistent with our observations about agricultural markets in sub-Saharan Africa.[45]

As for the existence of dual markets, Proposition 2 showed that, as long as $\tau < \tau_2$, some farmers would choose to sell to the BEM ($\underline{c} > 0$), for any equilibrium value of V^{TU} (including zero). Therefore, the free entry of traders would not affect the existence of dual markets. Intuitively, the existence of a dual market hinges on the negotiated price of an agricultural good in the BEM being sufficiently high. In that case, from a farmers' perspective, there exists a tradeoff between selling their produce to a trader at a high-price, but at the risk of incurring post-harvest losses, or selling their produce to the commodity exchange at a lower price, but at no risk of incurring post-harvest losses.

6. Conclusion

Agricultural markets in African economies typically operate in a decentralized manner. This trading system is associated with high levels of risk to farmers, who often incur post-harvest losses because they cannot find a trader to whom they can sell their produce. To address the inefficiencies of decentralized markets, several African countries have been considering the implementation of commodity exchange markets as a way of guaranteeing sales to farmers. In this paper, we formulate a search model to study the effects of the introduction of a commodity exchange market in a rural village where traders and farmers exchange in a decentralized market. We show that search frictions in the decentralized market generate two sources of inefficiencies. First, there are some farmer-trader matches which do not occur simply because farmers and traders do not find each other. Second, there are matches that do not take place because traders strategically reject the farmers that they are matched to because their costs are too high and it is in the trader's interest to re-sample the market. By introducing a commodity exchange market in this environment, we found that this market institution eliminates many of the economic disadvantages of the bilateral trading environment.

We find, however, that not all farmers benefit from the commodity exchange. Despite the advantages of the commodity exchange, there could still be dual markets where the commodity exchange co-exists with the bilateral trade environment. This occurs when the commodity exchange charges high transaction fees. In that case, high-cost farmers are left behind, as they find it more profitable to stay in the bilateral exchange market. This finding indicates that the existence of the commodity exchange might not be enough to bring such farmers to modern trading institutions, but that complementary policies such as reducing trade costs might also be necessary to incorporate such farmers.

One of the implications of the theoretical model is that many of the traders who were in existence in the bilateral trading environment will go out of business with the introduction of the commodity exchange. This is because the commodity exchange is able to provide intermediation much better than the traders. The commodity market, by creating a centralized market, is able to eliminate the lost farmer-trader matches. This was seen upon the introduction of the Ethiopian Commodity Exchange about a decade ago. In Ethiopia, these traders in the bilateral model were called Akrabis. For the most part, the Akrabis were wiped out. We suspect the same will be true in Ghana's case, as well as other African countries as they implement commodity exchange markets of their own. If the commodity market fees are too high, however, there may be high-cost farmers still in the bilateral trading environment.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jdeveco.2022.102867.

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[44] Empirically, the few papers evaluating the impact of commodity exchange markets based on the Ethiopian case find weak to no evidence of an impact on aggregate price volatilities. Tadesse and Guttormsen (2011) study the impact of the Ethiopian commodity exchange (ECX) on price volatility of maize and teff and Hernandez et al. (2015) the impact of ECX on price of coffee.

[45] Alternatively, one could imagine a model in which there is a supply curve of traders, where the mass of traders n_t^T is a function of V^{TU}. In this alternative model, because the mass of traders is a function of V^{TU}, the fixed-point strategy that we applied to solve for the equilibrium in our model would have to be reformulated.

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