From Bilateral Trade to Centralized Markets: A Search Model for Commodity Exchanges in Africa*

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Abstract

Several African countries have recently centralized their agricultural markets by launching a commodity exchange. What will be the impact of such a move? Who will be the winners and the losers? We develop a simple search model to study the impact of introducing a commodity exchange in a village economy where traders and farmers exchange on a bilateral basis. We study the efficiency gains from moving from the status quo to a trading regime where farmers have the option of selling their produce to a commodity exchange. We describe how the gains from trade are distributed between farmers, traders and the commodity exchange itself. We show that a dual economy where farmers sell both to the bilateral and the commodity exchange can exist in equilibrium, and that forcing all farmers to sell into the commodity exchange can make some farmers worse off.

Keywords: Agriculture, Bilateral Exchange, Walrasian Markets

JEL: J6, O2, Q1, F13

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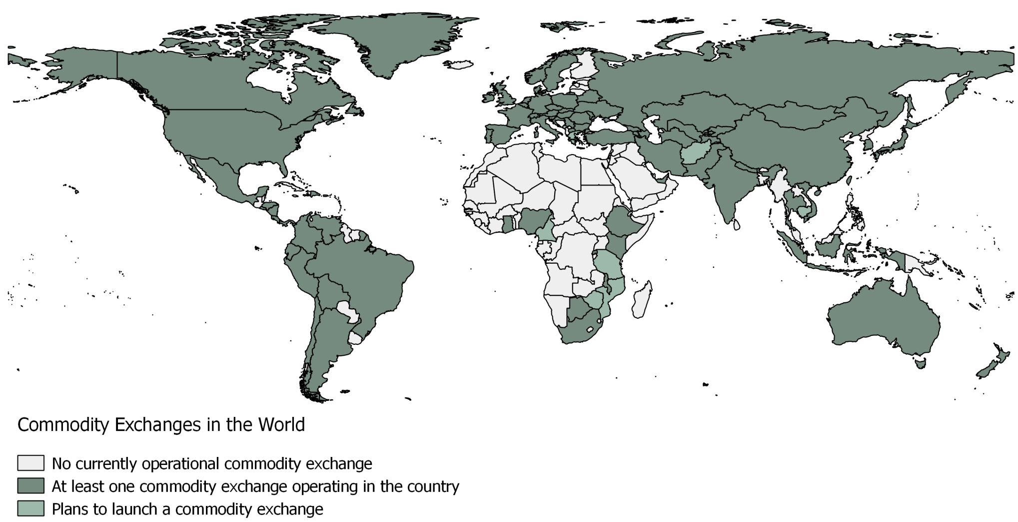
1 Introduction

The absence of modern trading institutions is perceived as an important cause of the large costs of trading in developing countries.[1] In most African countries, in particular, agricultural markets are still decentralized: farmers and traders search for a trading partner in local markets to trade on a bilateral basis. This trading environment, however, is expected to change in the near future. As shown in Figure 1, a few African countries have recently launched a commodity exchange and many are planning to follow in the next decade. In contrast to the decentralized system, in a market governed by a commodity exchange, transactions between farmers and traders occur in a predetermined location and are typically mediated by market makers who could be thought of as the Walrasian auctioneer as used in standard economics discourses.

Figure 1: Commodity Exchanges in the World as of 2019



Motivated by these recent developments, there have been a growing number of papers and policy reports examining the effects of commodity exchange markets on price fluctuations and price convergence between African regions.[2] There has been, however, comparatively less

[1]See Mezui, Rutten, Sekioua, Zhang, N'Diaye, Kabanyane, Arvanitis, Duru, and Nekati (2013), Rashid, Winter-Nelson, Garcia, et al. (2010) and Atkin and Donaldson (2015) for an analysis of trade costs in Africa.

[2]Several papers have recently studied the impact of commodity exchanges on the co-movement of prices across regions (Hernandez, Lemma, Rashid, et al., 2015; Andersson, Bezabih, and Mannberg, 2017; Minten, Tamru, Kuma, and Nyarko, 2014; Minten, Assefa, and Hirvonen, 2017; Gelaw, Speelman, and Van Huylenbroeck, 2017; Meijerink, Bulte, and Alemu, 2014; Sehgal, Rajput, and Dua, 2012; Tenderere and Gumbo, 2013). Also, see Mezui, Rutten, Sekioua, Zhang, N'Diaye, Kabanyane, Arvanitis, Duru, and Nekati (2013) and Rashid,

progress in terms of understanding how commodity exchange markets will affect the transactions between farmers and traders. In this paper, we contribute to this debate by providing a model to study the impact of introducing a commodity exchange in a village economy where farmers and traders exchange on a bilateral basis. In the context of our model, we address the following questions: How does a commodity exchange affect market efficiency? How are the gains from trade distributed between farmers, traders, and the commodity exchange itself? Under what conditions do decentralized markets co-exist with the commodity exchange?

We start our analysis by making several observations about the market structure of rural villages of developing countries. We illustrate our observations with a study area in Ghana and complement our description with micro-data from other sub-Saharan countries. In our context, farmers establish commercial partnerships with traders to whom they sell their produce on a regular basis. Traders, in their turn, sell farmers' produce to downstream markets. Traders are typically small, itinerant and constrained in terms of the number of farmers with whom they can trade in any given day or week, usually because of capital and other limitations. Since traders can make higher profits if they buy from farmers who can offer lower prices, they tend to be selective when they choose with which farmers they establish their commercial partnerships. Because traders lack information about which farmers have not yet sold their produce, searching for farmers to establish a commercial partnership takes time and resources.

In addition, we describe several features of a commodity exchange that is being progressively introduced in Ghana since 2018, called the Ghana Commodity Exchange (GCX). Specifically, we describe how the commodity exchange operates, the contracts that brokers trade, the location of official warehouses where farmers can deposit their produce for sale in the commodity exchange, who operates transactions on the market floor, and the fees charged by the commodity exchange to farmers who wish to sell their produce there.

Informed by our empirical observations, we develop a parsimonious model in which establishing commercial partnerships between small traders and smallholder farmers is a time-consuming activity. We consider a dynamic economy where farmers have heterogenous transport costs and traders face a homogeneous price at which they sell produce in downstream markets. In every period, a trader can pay a fixed cost to search for a farmer in his network of friends, relatives, and other traders. If a trader pays this search cost, she is matched to a farmer and the trade cost of the farmer is revealed to her. If a trader finds a farmer, she and the farmer engage in bargaining to determine the price at which she buys one unit of an agricultural good. If the trader and farmer reach an agreement, she forms a partnership with the farmer that is carried over to future periods until an exogenous shock breaks their partnership. If there is no agreement on the price (or no bargaining at all), she has to pay the search cost again to find

Winter-Nelson, Garcia, et al. (2010) for policy reports documenting the experience of developing countries with the implementation of commodity exchange markets.

another farmer in future periods. If farmers are not matched to a trader by the end of a period, they either consume their own produce or incur post-harvest losses.

After presenting this search and bargaining environment, which we refer to as the bilateral exchange market (hereafter, BEM), we use a fixed point argument to prove the existence of an equilibrium and show that it is unique. In addition, we derive the aggregate supply of agricultural produce and show that there exist two sources of inefficiency in this economy. The first one comes from the randomness of the search process. In every period a mass of farmers who could potentially generate positive market surplus—i.e., whose trade costs are below the price of a unit of the agricultural good in the local market—are not matched to any trader due to the randomness of the search process. The second one comes from the fact that traders strategically reject bargaining with farmers in order to wait for better matches in future periods. Specifically, we show that there exists a mass of high-cost farmers whose transportation costs are below the price of produce in downstream markets, so that they could generate a positive market surplus for the economy, but who are still rejected by traders who opt to forgo current gains for the chance of being matched with low-cost farmers in future periods.

We next give farmers the option of selling their produce to a commodity exchange market (hereafter, CEM) instead of only having the option of waiting to be matched with traders. If farmers choose to sell to the CEM, they can immediately sell their produce and avoid the risk of post-harvest losses, but they have to pay a fee that creates a wedge between the price of their produce in the CEM and the price of their produce in downstream markets. We show that, in that case, the traditional BEM tends to coexist with the CEM, as some farmers choose to sell to the CEM and others to the BEM. In addition, we demonstrate that the benefits of immediacy through the CEM are larger for lower cost farmers. It is therefore the low cost farmers who opt to sell their produce to the CEM. High-cost farmers still opt to wait for traders to sell their produce in the BEM.

When the conditions for the coexistence of the CEM and the BEM hold, the implementation of a CEM generates important distributional gains from trade. First, some farmers who choose to sell their produce to the CEM are better off, since they avoid the risk of post-harvest losses. Second, as some of these farmers move to the CEM, some traders may lose because they have to operate in a BEM where the remaining farmers have higher costs. Third, farmers who stay in the BEM are potentially also better off. Even though they remain in the BEM, they are potentially less likely to be strategically rejected by traders, and they also could have a higher bargaining power when negotiating prices—that is because traders have fewer farmers competing for their attention.

We also study the aggregate welfare. We show how aggregate welfare rises upon the introduction of the CEM due to the elimination of the post-harvest losses. We indicate how the CEM affects the bargaining and negotiations with the lower numbers of high cost farmers, potentially increasing the willingness of traders to accept higher prices, and expanding the set of (high cost) farmers who are matched and therefore sell their produce.

Using our framework, we examine two types of implementations of commodity exchanges that have been widely discussed by policymakers: the full and the partial mandate. In partial mandate, as in the new Ghana Commodity Exchange, farmers have the option of selling to the CEM or to BEM, which is the case in which both types of regimes might coexist. In full mandate, as in the Ethiopian Commodity Exchange, the government forces all farmers to sell their produce to the CEM, which guarantees a minimum volume of transactions in the CEM, but makes some farmers with high costs worse off since they would have chosen to sell in the BEM.[3] Hence, through our model, we show how the incentives for the formation of informal and illegal markets emerge, an important concern among policymakers.[4]

Lastly, we close our paper by considering several extensions to our main model and how they affect two of our main results: the coexistence of market institutions and the strategic rejection of farmers by traders. We discuss the many simplifying assumptions of our model. In particular, we discuss the importance of our assumptions about the number of matches a trader can have within a period and the questions of exogeneity versus endogeneity assumptions used in modeling the search intensity of traders. We also discuss the consequences of having different sources of price volatility in our model, the assumptions of risk neutrality versus risk aversion of farmers, and our assumptions on how we characterize the entry of traders into a market.

Related Literature. Our paper relates to different strands of research. First, our model predicts the co-existence of the CEM with decentralized markets, which relates to the concept of dual markets that has a long history in development economics. As elegantly and forcefully noted by Gollin (2014), it was the core of the analysis of Arthur Lewis, considered by many as the father of development economics, in his seminal paper Lewis (1954). Arthur Lewis emphasized a dual economy where there was a modern sector living side by side with a traditional sector. In our paper we may think of the commodity exchange market as the modern and the Bilateral Exchange Market as the traditional. Our main theorem shows how the two can live side by side in equilibrium.

Within the more recent development literature, bargaining models have been used in papers

[3]A common risk that commodity exchanges face is the lack of sufficient transaction volumes. In that case, the capacity that the commodity exchange has to guarantee the delivery of a product is limited. Furthermore, commodity exchanges have large fixed costs, but low marginal costs of individual transactions. If there is not enough volume transacted on the floor, commodity exchanges may not generate sufficient revenues to pay for their fixed costs. The risks of insufficient scale are higher when a commodity exchange coexists with a decentralized market, which is the more common system. In some cases, to minimize this risk, governments opt for a fully mandated system, where the government bans some types of market transactions from taking place beyond the commodity exchange floor.

[4]See more detailed discussions about the tradeoffs between partial and full mandate systems in Anderson and Baulch (2017).

looking at the impact of price information, usually transmitted via mobile phones, given to farmers who sell their goods to traders, including Hildebrandt, Nyarko, Romagnoli, and Soldani (2015), Aker (2010), Aker (2008), Courtois and Subervie (2014) and Svensson and Yanagizawa (2009). We contribute to this literature by examining how the introduction of a commodity exchange affects the transaction price between farmers and traders.

Our theoretical framework builds on two early models of decentralized markets, Rubinstein and Wolinsky (1985) and Rubinstein and Wolinsky (1987). The paper Rubinstein and Wolinsky (1985) formulates a search and bargaining model with two sided markets. Their model has two types of agents: buyers and sellers. Agents in each side of the market are homogeneous. There is no strategic rejection and, as in our paper, matching probabilities are fixed. The paper Rubinstein and Wolinsky (1987) develops a search and bargaining model with three types of agents: buyers, sellers and middlmen. Agents in each side of the market are again homogeneous and matching probabilities are fixed in steady state. In our paper, we use tools developed in these two papers—particularly on the Nash Bargaining outcome. We have essentially two sided markets, farmers and traders. (In principal, we could think of a third type of agent, the final buyer, however this is trivial as the price in downstream markets is fixed). In contrast to these papers, we have strategic rejection, heterogeneous agents in one side of the market, and the coexistence of decentralized and Walrasian markets.

By applying the tools from these earlier models, our paper relates to applications of search models to asset markets. A key reference here is Duffie, Gârleanu, and Pedersen (2005), who develop a search and bargaining model for over-the-counter markets. They have investors with and without assets, and with low and high costs of holding assets. Investors with high costs of holding assets search for investors with low costs to buy their assets and vice versa. Lagos and Rocheteau (2009) and Afonso and Lagos (2015) extend the model in Duffie, Gârleanu, and Pedersen (2005) by allowing for more flexible asset position. In these papers, when agents are matched they bargain over the price of an asset and there is no strategic rejection. In our model, agents instead bargain over a contract that includes multiple transactions over time and there is strategic rejection.

The interactions between farmers and traders in our model also relates to applications of search models to labor markets, including Pissarides (1985) and Mortensen and Pissarides (1994). In these two papers, workers and firms search for each other and, when matched, they bargain over wages.[5] To the best of our knowledge, this literature has not modelled the co-existence of walrasian and decentralized markets, in part because Walrasian markets are extremely rare in labor markets. We believe that our framework has applicability for cases

[5] See Rogerson, Shimer, and Wright (2005) for a throughout review of the literature on the applications of search models to labor markets. Another common approach in this literature is to have search without bargaining. In this case, workers and firms post wages ex-ante and workers search until they find a firm that offers a wage that is above their reservation wage.

where Walrasian markets can coexist with decentralized ones, particularly for markets where agents negotiate loans of assets such as machinery and warehouses.

We know of few papers that study the co-existence of walrasian and decentralized markets. One reference is Rust and Hall (2003) who introduce a Walrasian market into the middlemen model formulated by Spulber (1996). They have three types of heterogeneous agents in their model, buyers, sellers and middlemen. The structure of their model is substantially different from the earlier models of search and bargaining developed by Rubinstein and Wolinsky (1985) and Rubinstein and Wolinsky (1987). In Rust and Hall (2003), there is no bargaining and prices are posted ex-ante.[6] Another reference is Miao (2006), who studies the introduction of a commodity exchange in a search an bargaining environment. Different from his paper, we have strategic rejection and we study the distributional gains from trade. A common feature in these two papers is the existence of a constant flow of new agents in the economy who leave the market after being matched. In our model, we instead have a fixed mass of agents on each side of the market (some matched and others not) who bargain over contracts that include transactions over multiple periods of time.[7]

Lastly, we contribute to recent research in trade using tools from search theory (Antras and Costinot, 2011; Allen, 2014; Startz, 2016; Antràs and Staiger, 2012; Krishna and Sheveleva, 2017; Atkin and Donaldson, 2015). Closest to our paper is Antras and Costinot (2011), who examine the gains from trade between countries in a dynamic search model where traders and farmers establish contracts to exchange agricultural commodities. In their model, farmers are homogeneous and can produce one of two goods. In our model, we have a single good, but farmers have heterogeneous trade costs.[8] In addition, while in their framework farmers can only sell their produce to traders, here we study the effects of giving farmers the option of selling their produce to a commodity exchange. Another recent paper in this area is Atkin and Donaldson (2015), who show that imperfect pass-throughs contain information about market structure and provide estimates of imperfect-pass-throughs of manufacturing goods in Ethiopia and Nigeria. In the agricultural context, Fafchamps and Hill (2008) document imperfect pass-throughs for coffee in Uganda. Here, we model a specific micro-economic mechanism that generates imperfect pass-throughs: the search costs of decentralized agricultural markets.[9]

[6]See Rogerson, Shimer, and Wright (2005) for a discussion about models with ex-post and ex-ante price setting.

[7]In addition, Gehrig (1993) formulates a static model with random match and bargaining. Since his model is not dynamic is does not account for the strategic considerations that we examine in our model.

[8]Allen (2014) and Krishna and Sheveleva (2017) also formulate search models to analyze the transactions of goods in developing countries, but different from Antras and Costinot (2011), their models are based on ex-ante posting of prices and no-bargaining between the two sides of a market.

[9]In our model, we assume perfect competition in downstream markets where traders resell their produce, but search markets in upstream markets where traders buy their produce from farmers. This market structure is consistent with results from Fafchamps and Hill (2008), who document a large pass-through between international and wholesale prices, but a small pass-through between international and farmers' price in Uganda.

2 Case Study in a Rural African Market

This section describes the *status quo* farming environment and market structure of an agricultural market of a village in a typical sub-Saharan African country. We document the market structure in our study area based on research we carried in Ghana from 2015 through 2019.[10] We complement our description using recent micro-data from the World Bank for other sub-Saharan countries. This section also describes key characteristics of the commodity exchange recently introduced in Ghana, the Ghana Commodity Exchange (GCX). Lastly, we close this section with a brief discussion of how we map our observations to the structure of the model, which we develop in Section 3.

2.1 Our study area

We will focus on the crops which are traded on the Ghanaian commodity exchange: primarily maize, soya and rice, with some sesame and sorghum. Our focus will be on smallholder farmers in Ghana. They form the bulk of the farming in the country, both in terms of the numbers of people involved and in terms of the volume produced.

We begin with general observations about the market microstructure in these areas. What we describe here is the *status quo* environment before any commodity exchange is introduced. Our pilot study area is a portion of the central part of Ghana, in the Kumawu Traditional area (the Sekyere Kumawu and Sekyere Afram Plains parliamentary districts plus small amounts of 2 or 3 others surrounding these districts). This area covers around 5000 square kilometers, approximately 2% of the land mass of Ghana. As of the most recent publicly available census, our pilot study area has around 120,000 inhabitants, making it a relatively sparsely populated area.

2.2 Main observations

Below we list key observations about the study area. The next section describes the key agents in the market.

1. **Land.** For smallholder farmers, land issues are not currently a major constraint on their production. When such farmers seek to marginally expand their production, there is typically vacant land within their properties or informal markets to rent their neighbors piece. But land can sometimes be an issue for large-scale farming.

[10]Nyarko is grateful to the International Growth Center and Anonymous donors for the research grants that enabled this research to take place.

2. **Labor.** Farmers use their own time and labor on their farms and also hire laborers. The labor is required to clear (or "weed") the farms and also to carry produce from the interior of the farm to the farm gate. As of the time of writing it costs around GHS 20 (around US $2) per day for these laborers, who in the local parlance are called "by day laborers." These by day laborers help with cutting the weeds, harvesting or spraying.[11]

3. **Transport cost.** The transport sector involves high fees for moving produce for farmers, relative to their revenues. Yet those with the produce can have them transported to local markets for the most part. These fees are commonly perceived by farmers as surmountable so long as they find customers to sell their produce to.

4. **Agricultural inputs.** Fertilizer use is extremely low. Farmers indicate to us that they know that fertilizer use is important, but they claim that it does not make economic sense to invest in fertilizers. Some farmers are afraid of spending money on fertilizers, perhaps with borrowed money, only to see the markets collapse on them at harvest time. Other farmers complain that they have liquidity or cash constraints which prevent them from purchasing fertilizers. Those farmers also do not go to the banks for loans because, again, they fear the consequences of a market collapse at harvest time when they have no money to repay their loans.

5. **Technology.** Advanced technology is non-existent and given current prices, the use of such technologies is probably not optimal at this time and at the scale of production of the farmers. There are no irrigation schemes among the smallholder farmers we worked with. In a national survey we conducted among some 1200 farmers, only one group in our study, a group producing maize, hired the services of a tractor. The vast majority of farmers use only one implement in their farming, the cutlass.

6. **Finance.** Many farmers indicated that financing is a major issue. They indicated that with more capital they could expand their farms. When asked why they did not go to the bank for a loan, they frequently say that this is because of fear of not getting a good price for their output and then falling into debt. Farmers often take loans from traders in exchange for selling their goods to the trader at harvest time. However, farmers say that they do not like this arrangement. This is because traders would dictate a price to them when the harvest came, thereby extracting an exceptionally high implicit interest rate on the loan.

[11] An alternative method of contracting labor is by acreage, which costs approximately GHS 150 (around US $30) per acre. The laborer given that contract will be required to work on that area to get paid and will be paid proportionately to the total acreage worked on.

7. **Demand.** Lack of sustainable demand for farmers' crops seems to be the biggest constraint to the development of the smallholder agricultural sector. Farmers complain a lot about not being able to get buyers for their produce. When farmers are asked why they do not use fertilizer or advanced technology or take bank loans, the answer almost always seems to involve the lack of sustained markets for their goods. Indeed, commodity prices are erratic and farmers do not always know what the prices are going to be for their goods.

8. **Storage.** Warehouses and storage facilities are non-existent for many crops of many farmers. There are a variety of techniques that farmers employ which amount to implicit storage. For example, yam farmers keep the yams in the ground until they are ready to sell. Other crops are left unmatured and treated with chemicals to make them flower quickly when there is a need to sell these. Still, despite farmers efforts to minimize potential losses, there are large post-harvest losses in the region.

2.3 Key agents in the market

Farmers. The main crops grown by farmers in our area are yam, plantain, cassava and maize. Some farmers also grow cocoa, but cocoa is a cash crop that is managed by the government. Some less important crops, by volumes and revenues, include garden eggs (eggplant), tomatoes and other vegetables, cocoyam, groundnut and rice. Very few of the farmers we interacted with use any kind of mechanization. The main implement used by farmers is the cutlass and nothing else. The cutlass is used to clear weeds, make holes in the ground to insert seeds, etc. Farmers do pay attention to the seeds and the methods of planting. They obtain seeds from the previous harvest or through nurseries in neighboring communities. They get a lot of advice from the government Ministry of Food and Agriculture (MoFA) extension agents. Farmers use chemicals, namely weedicide, to keep out unwanted grass and shrub. Some farmers complained about pests affecting their crops. They also complained that because of insufficient funds they are unable to engage in pest control and use herbicides. Farmers indicate that they are cash-constrained and almost never purchase the required amount of fertilizers as instructed by the government extension agents. When they "get some good money they will invest in fertilizers", they told us, otherwise, they take their chances on the over-worked soil on their farms.[12]

In our study area, as in many sub-Saharan African nations, farmers live on their tribal

[12]Some farmers indicated that animals destroy their farms. None of the farms we studied had any fences or barriers demarcating and sealing off their farms. The region is currently being invaded by cows roaming the bush led by itinerant or nomadic pastoralists. These pastoralists travel from the Sahel areas, particularly those affected by climate change. They head south to less affected areas. This has resulted in many violent clashes recently between the owners of these roaming cowherds and the indigenous people farming on the same contested lands. These issues have been documented in our study area and in surrounding areas.

lands. The people farming in an area are often from families that have been there for centuries. Geographic mobility is not that high among farmers growing the crops we study. There is a great difference in transport cost between being close to the main road or far from one, or being relatively close to one but having no access to that road. Even a few miles distance away from the main road through dense forest and steep hills or difficult river crossings can make a huge difference in trade costs. Different crops are grown in different areas. However, within an area that, for example, grows maize there could a lot of variation in production costs due to the reasons just mentioned. This difference would exist even when the geographic distances are relatively small and agroclimatic conditions are similar.

Traders. There are many different types of traders. Most of these were women. They are intermediaries of various sizes, but most of them are very small. Many of the small traders take goods from farmers and send them to regional markets which are around 1 to 2 hours away by car. These small traders are the majority of those who live in our study area. Our farmers occasionally interact with big traders who collect goods from farmers for sale in Accra. One farmer mentioned, "I trade with 2 people from Accra and I sell to them on Thursdays." One interesting feature of traders' activities caught our attention. Sometimes traders "buy the farm," as they say in the local parlance. What this means is that the farmer and trader negotiate for a certain amount of the farmer's farm—for example, 2 acres of a farmer's plantain farm. The trader will pay the farmer a price and then the trader will be responsible for hiring the laborers to harvest the produce (the plantain in this example) and pay for the transportation of the produce from the farm to village. This is an interesting way in which the farmers deal with their lack of liquidity or their lack of ability to pay upfront for their labor and transportation costs. The traders who buy from farmers in this manner represent the principal type of partnership with farmers in our study area. When we formulate our model, these small traders are the ones that we have in mind.

The trading activities of small itinerant traders are often, but not always, coordinated by a "market queen" (Clark, 1994), who is usually an elected representative of traders in regional market for a given staple. Market queens coordinate the market space: for example, they determine how many trucks can enter in the urban area, who can become an itinerant trader in the area, settle disputes and negotiate with local governments (Lyon, 2000). Another barrier that traders face to enter in a market is that they require many years of experience to consolidate their activities. Traders typically build long-standing trading relationships with farmers. These relationships operate as informal contracts based on farmer's ability to deliver the produce in time and trader's ability to sell their produce in central markets.[13]

[13]Lyon (2000) documents the many ways through which these relationships are built. For example, through gifts, attending funerals and traveling to villages to participate in the trading of produce.

Traders and local markets. One interesting feature of markets caught our attention and may be a local response to the various constraints faced by market participants. In any one market town, markets operate once a week. The days of the week differ in different towns. For example, the market town Bodomase operates on Fridays and the market in Juaben on Wednesdays. By having markets open once a week, traders are able to aggregate produce from more farmers and get the volume needed to make their operations scale up. All traders would, for example, converge on Bodomase on Fridays. The farmers in that area will farm most of the week, then collect all their produce on Thursday night at their house or in a local storage area and have them ready for traders to inspect and hopefully purchase early in the morning on Friday. In some local markets, we did hear of price fixing by traders. We were told of some instances where traders agree in advance on what the price of a particular crop should be. We did not hear of this for all crops, and this effect seemed to be dying down. One farmer, a woman, said to us "they used to have fixed prices for tomatoes, however that process has died.". We did not find much of this price fixing occurring in the big cities and towns.

Despite small traders' efforts to scale up their operations, they tend to be cash constrained, which limits how much they can buy in advance from farmers before weekly markets. In addition, transportation technologies deter how much they can bring to local markets. In our study area, most traders rely on small vehicles. In other sub-saharan regions, transportation technologies can be substantially worse. For example, in Ethiopia and Tanzania, more than 80% of farmers have their produce transported on foot, on a bicycle or with a draft animal, and less than 18% of farmers have their produce transported on a truck, bus or a minibus (see Appendix Table A.1).

(We did not document in our research buyers who buy for their own consumption (like the poultry farmers who purchase maize) - they are similar to the final buyers in regional markets that traders sell to. Almost all of the buyers in our study area resell their produce in retail markets, therefore we consider them all to them as traders.)

Traders, information asymmetries and search. In general, farmers do not know the prices of goods in major markets and do not arbitrage price differentials across time. One question we posed was this: why don't the farmers just call a friend in the market in the main city to ask for the current prices? We found out in research that the farmers did not have friends who had access to the market prices. Since prices moved around so much, even if they knew someone in the city, it would probably be hard to ask that person to go to the market every week just to check prices.

In addition to limited information about prices, modern storage methods are too expensive for farmers to afford, which constraints their ability to make strategic decisions about the timing of their sales. Data for other sub-saharan regions indicate similar constraints in Ethiopia, Tanzania and Malawi. Appendix Table A.1 shows that more than 80% of farmers in these countries use traditional storage methods. When asked about the reason why they store their produce, the majority of farmers declare that they store for their own consumption, and only 3 to 8 percent of them say that they store to capture higher prices in the future.

Farmer-trader matches and post-harvest losses. Farmers typically wait for traders to buy their goods at a good price and do not sell their produce in local markets themselves.[14] They told us often that they would be at their farm gate looking for or waiting for traders but would not have any who visit them. These farmers often live in faraway and remote areas where trade costs are high. Similarly, we spoke to many traders who told us that it is hard to find farmers to trade with. In particular, it seemed as if there could be viable matches if only the traders wanting goods and farmers with the goods to sell could locate each other. We also remark here that we found many situations where the farmers would negotiate with traders on the appropriate price to sell their goods at and not reach an agreement.

2.4 The Ghana Commodity Exchange (GCX)

According to the Oxford Dictionary of Economics, a commodity market is "A place or institution through which commodities are traded. Markets were originally places or buildings, where traders could come together, which facilitated comparisons of price and quality. [. . .] Commodity markets include both spot markets, where goods are traded for immediate delivery, and forward and futures markets, where prices are agreed in advance for delivery at various dates in the future." In November of 2018, Ghana launched the first commodity exchange market in West Africa, called the Ghana Commodity Exchange (GCX), to operate as a spot market and trade crops from various locations.[15] In its inauguration speech, the president of Ghana Nana Akufo declared that the goal of GCX is to reduce post-harvest losses and benefit farmers by securing storage for their harvest.

A commodity to be traded in the GCX is a contract that is indexed by a crop (e.g., maize, or soya), a quality grade (grade 1 is the best grade, grade 4 is the worst), and a warehouse location (e.g, in Tamale or Sandema in the north of Ghana). For example, there could be trade in White Maize grade 1 sitting at the Tamale warehouse – this contract would be given the symbol TAWM1, where TA stands for Tamale, the WM stands for White Maize, and 1 stands for grade 1. There are currently 9 warehouses, all across the country.

At fixed times during the day, e.g., Monday at 1pm GMT, there will be trade in a contract, e.g., TAWM1. Sell side brokers who represent farmers with maize (TAWM1) to sell will post

[14] This is the case in other sub-saharan regions. As discussed in Fafchamps and Hill (2005), almost all of coffee growers in Uganda sell their produce at the farm-gate.

[15] The only derivative-based exchange in Africa is the South African Futures Exchange (SAFEX).

offers on the market. Buy side brokers who represent people who want to buy the maize (e.g., poultry farmers) will similarly post ask prices on the platform. The system then matches buyers and sellers with compatible prices and trade takes place.[16] Once this is done, the trade is complete. The buy side broker pays for the good by transferring money into what is called the Central Depository, which is run by the exchange. The Central Depository in turn will transfer that money to the broker on the sell side who then transfers the money to the farmer who deposited the crop in the first place.

GCX is owned by the national government and any farmer may pay a fee and sell their produce at one of the official warehouses. These official warehouses are located in regional centers, close to the location of regional markets. Therefore, the costs of transporting farmers' good to the nearest warehouse tend to be similar to the cost of taking a good to a regional market. The fees are charged to cover the operation costs of the commodity exchange and are proportional to the volume of goods deposited by the farmer in the warehouse. The basic fee amounts to up to 10% of the price of the good. Adding drying, fumigation and cleaning increases this fee up to 40%. For the 40 or so farmers we interviewed in our pilot study, the average fee paid to the commodity exchange was approximately 17%. In the Appendix Section E, we included a few receipts that we have collected of actual transactions between farmers and the GCX for reference.

Lastly, due to the technical nature of the work, brokers in the commodity exchange need a minimum of Bachelor's degree. In particular, they are required to have experience in Financial Management and IT. There is a lot of technical information that they need to grasp, and very quickly. They need computer skills, business management, understanding business models and trading trends, and risk management. The small itinerant traders operating in the traditional markers are, therefore, unlikely to become brokers in the commodity exchange, at least in the short run.

2.5 From our observations to the model

Our observations indicate that search costs (traders finding farmers) in our study area of Kumawu are substantial. The lack of information about farmers forces traders to spend a substantial amount of their time and money to the activity of trading itself, such as contacting suppliers and inspecting farms. Once traders find farmers, they negotiate the price of the produce, since there is not a common pre-established price in their market. Traders are generally small and, due to cash-constraints and poor transportation technologies, can not buy from several farmers in any given week or day. Because search takes time, many farmers are not matched to a trader

[16] Acceptable price pairs are those where the buy side broker's bid price exceeds the sell side brokers ask price. After a match the system generates an acceptable price, the midpoint. In practice, however, buy side brokers look at the price list and post and accept prices of sell side brokers that they find agreeable.

and end up using their produce for subsistence consumption or incur post-harvest losses. In general, farmers do not to sell themselves their produce in local markets.

Our goal in the next section is to model the transactions between small traders and smallholder farmers who are currently producing corn and yam, since these are the transactions that will be directly affected by the introduction of the GCX.[17] This commodity exchange market will give farmers the option of selling their produce to the CEM at any time, avoiding the risks of post-harvest losses if they are not found by any trader. As highlighted by the president of Ghana in his speech during the inauguration of the GCX: "Ghanaian farmers will gain access to secured storage for their harvest and good warehousing management practices, thereby improving their take-home sales." Our model will be designed to capture precisely this type of loss.[18]

For tractability, we adopt a few simplifying assumptions to focus on this particular view of the benefits of the commodity exchange, which is related to search costs and post-harvest losses. Specifically, we adopt two assumptions about farming production based on our empirical observations. First, given that the scale of production is extremely small and that there is little to no use of agricultural inputs among smallholder farmers, we assume that land is the only factor of production. Therefore, our results should be interpreted with caution as there are many ways in which they may understate the positive impact of the commodity exchange. For example, while in the short run technologies may be fixed at the current low levels of adoption, in the long run a CEM might encourage the adoption of new technologies. Second, we do not model credit markets. Our observations suggest that the low uptake of loans is partially a result of the uncertainty generated by the lack of consistent demands, as could be obtained from a centralized market. With the implementation of a commodity exchange, farmers might increase their loan uptake—although there are certainly different views on this future potential of the exchange (see Jayne, Sturgess, Kopicki, and Sitko (2014)).

3 The Bilateral Exchange Market (BEM)

This section develops a parsimonious search and bargaining model to describe an agricultural market such as the one represented by our study area. This model will help us understand the impact of the introduction of a commodity exchange which is beginning to take place in this community. We organize this section in three parts. First, we characterize the economy at the

[17] Larger farms producing cocoa, for example, already sell their produce to the government or directly to international trading companies.

[18] In addition, the president declared that "Most often, there are no formal contractual agreements in place, resulting in trade disputes between our farmers and buyers which undermine our marketing system. These are some of the challenges we are aiming to address." In our model, we will be capturing these types of informal contractual agreements between farmers and traders by assuming that they establish trading partnerships that endure for multiple periods of time.

status quo when there is no commodity exchange, which we call the pure bilateral exchange market (BEM). Second, we define the equilibrium and prove the existence and uniqueness. Third, we discuss the gains from trade and the aggregate supply of agricultural produce in the community. In the next section, we introduce a commodity exchange market into the economy.

3.1 Economic Environment in a Pure BEM

Consider an economy with two types of agents, farmers (F) and traders (T). This economy operates over time. The time dimension is discrete. Farmers and traders live forever and are risk-neutral. In each period, farmers produce one unit of a non-storable agricultural good. They can sell their agricultural good to a trader who then take their produce to a regional market, where agricultural goods are sold at a price P. (We often refer to P as the "Accra" price.) We denote by c the costs that farmers incur in every period that they take the agricultural good from their farms (which are usually in remote areas) to the farm gate (roadside) or to local markets. We can think of these as transport costs from the farm (which could be in the "bush" to the farmgate by the road), although it is easy to think of c also incorporating other production costs. There is a unit mass of farmers with costs coming from a uniform distribution between 0 and c^{max}, such that the density of farmers equals to $g(c) = 1/c^{max}$, and an unit mass of homogeneous traders. We normalize to 0 the price that a farmer receives for his produce if they do not sell it to a trader. This can be be considered as a post-harvest loss. We think of a scenario where the farmer has an abundance of food for subsistence (home consumption) and the crops we are studying are primarily for sale, with those for home consumption coming from their gardens or easily obtained on an almost daily basis from their farms.

In this economy, farmers and traders can either be unmatched or be matched in a trading partnership with each other. Traders who are interested in forming a new trading partnership can pay an upfront cost κ, in every period they are searching, to contact their network of relatives, friends, and other traders and search for an unmatched farmer. Traders need to find farmers who are "real" and reliable, since legal enforcement mechanisms are weak and fraud is a real possibility. Farmers, on their turn, focus on production and do not search for traders. When a trader finds an unmatched farmer, the cost parameter of the farmer c is revealed to the trader who then decides whether to negotiate the price of produce $p(c)$. Farmers are matched with a trader with exogenous probability μ^F. Traders are matched with a farmer with exogenous probability μ^T.[19] If they reach an agreement, they leave the network and start a trading partnership that is exogenously broken with a probability β. Once the farmer leaves the

[19]In a typical search model, the probabilities μ^F and μ^T are determined by a matching function - which generally adopts a constant returns to scale form—that depends on the mass of unemployed farmers and traders in the market. Here, assuming that μ^F and μ^T simplifies substantially our analysis. In the appendix, we provide additional justification to our approach.

network, she is replaced with another farmer with the same cost c. This replacement guarantees that traders' and farmers' decisions are replicated in every period. In this environment, search is costly for traders for two reasons: first, because traders have to pay an upfront cost of κ to search; second, because if traders do not find a farmer during the search process, or if they do find a farmer but choose to reject negotiation, they have to wait until the next period to search again.

By way of summary, we note that the cost c borne by farmers is incurred each time the farmer's goods are sent to the farmgate to the trader, but is not incurred if there is no match with a trader. Also, the trader cost κ is incurred in each period the trader is unmatched and searches for a new farmer; this cost is not incurred if the trader is matched with a farmer.

To characterize the farmers' and traders' decisions, we have to account for the fact that agents consider the outside option of waiting to find a better trading partner in future periods. In particular, the gains from trade for them is the value of establishing a partnership and being matched against the value of being unmatched and waiting for a potentially better trading partner. For a farmer, the cost of waiting is to lose the opportunity of selling his or her produce to a trader by $p(c)$. For a trader, the cost of waiting is to lose the opportunity of selling an agricultural good right in the beginning of the next season at local market by a price P.

Let V^{FM} and V^{FU} be the values of being matched and unmatched for a farmer, and V^{TM} and V^{TU} the respective values for a trader. The value functions of farmers and traders are given by the following Bellman equations:

$$
\begin{aligned}
V^{FM}(c) &= p(c) - c + \delta\left\{\beta V^{FU}(c) + (1-\beta)V^{FM}(c)\right\} && (1)\\
V^{TM}(c) &= \max\left\{P - p(c) + \delta\left\{(1-\beta)V^{TM}(c) + \beta V^{TU}\right\}, V^{TU}\right\} && (2)\\
V^{FU}(c) &= \delta\left\{\mu^F V^{FM}(c) + (1-\mu^F)V^{FU}(c)\right\} && (3)\\
V^{TU} &= \max\left\{\delta\left\{\mu^T \int V^{TM}(c)g(c)dc + (1-\mu^T)V^{TU}\right\} - \kappa, 0\right\}, && (4)
\end{aligned}
$$

The first equation describes a farmer with cost c who is matched with a trader in the beginning of the period. It says that the value function of the matched farmer with cost c is equal to what the farmer gets in the first or current period, the price $p(c)$ minus the cost c, plus the discounted value of the future utility value. The future value is determined by whether the farmer is unmatched or matched in the next period, $V^{FU}(c)$ or $V^{FM}(c)$, events that occur with probabilities β and $(1-\beta)$ respectively.[20]

[20]Notice that we impose a one period delay between production and trade. We allow this delay to capture the fact that there are seasons and a natural rhythm to the farming cycle. Here, we think of each period as a whole cycle of a crop. During the planting and growing seasons, farmers meet with traders and negotiate prices to guarantee their sales when the harvest season arrives. When farmers are done harvesting and cleaning their crops, they hand their produce to traders. Many farmers sell all their crops in one go. Even if it is not in one go, it is over a very short period of time.

The second equation (2) pertains to a trader who is matched with a farmer with cost c in the beginning of the period. That trader has to decide whether to trade with that farmer (the left hand term in the bracket after the max) or else to walk away (we call this strategic rejection) and become unmatched, with a value function of V^{TU}. If the trader does trade with the matched farmer, then the trader makes in the current period a profit equal to the difference between the big city price P and the bargained price $p(c)$ plus the discounted value of the expected return to being matched with a farmer of cost c.

The third and fourth equations, (3) and (4), pertain to the unmatched farmer and unmatched trader. In each case they receive 0 in the current period, their future returns are discounted by δ and they receive the expected return to being matched and unmatched in the next period, events that take place with probabilities μ^F and $1-\mu^F$ for the farmer and μ^T and $1-\mu^T$ for the trader. The trader pays a search cost of k when unmatched and beginning a search.

Let η denote the bargaining power[21] of the trader and define $\phi = (1-\eta)/\eta$ as the power of a farmer relative to a trader. In particular, at each value of c where there is trade between the farmer and the trader, we have the relation that the surplus going to the farmer is ϕ times the surplus going to the trader:

$$V^{FM}(c) - V^{FU}(c) \quad = \quad \phi \left\{ V^{TM}(c) - V^{TU} \right\}. \tag{5}$$

We stress here that the equation above will be required to hold only for those values of c such that both parties, the farmer and the trader, want to trade.[22] In particular, the equation above will only be required to hold when the maximum on the right hand side of equation (2) and also of equation (4) each occurs in the first term and not in the second term inside the maximum operator. If we consider as parameters δ, β, μ^F, μ^T, c and P, the equations (1)-(5) are a system of 5 equations in the 5 unknowns $V^{FM}(c)$,$V^{TM}(c)$,$V^{FU}(c)$, V^{TU}, and $p(c)$. It is easy to see that we can in principle get solutions for the 5 unknowns in terms of the parameters. We next define the market equilibrium in this economic environment.

[21] η is the Nash bargaining weight so that any surplus (less outside options) is shared in the proportions η for the trader and (1-η) for the farmer.

[22] Notice that in equation (5), even though the bargaining parameter ϕ is fixed, bargaining outcomes are still a function of outside options and so are "endogenous." The outside options define the size of the "gains from trade" that needs to be divided among the players or our bargaining game – the trader and the farmer. After the gains from trade have been defined by the outside options, the split of the gains is defined typically by the different time discount rates in models like that of Rubinstein and Wolinsky (1985). In particular, the final negotiated price between traders and farmers in our case is the outcome of a bargaining process in which farmers are actively choosing whether to leave the negotiation or not.

3.2 Definition of Equilibrium in pure BEM with positive flows

Definition 1. Fix a model with parameter set $\Omega = \{c^{max}, P,\, c,\, \beta,\, \delta,\, \kappa,\, \mu^F,\, \mu^T,\, \phi\}$ and cost parameters c uniformly distributed in the set $[0, c^{max}]$. The equilibrium is a pricing function $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$, $V^{TU}(c)$, V^{TU}, defined via the following two stage game.

1. In the first stage, a pricing function $p(c)$ is set within the BEM. Conditional on the behavior of farmers described below, no individual trader has an incentive to offer a price different from $p(c)$ to a farmer with cost function c.

2. In the second stage, each farmer with cost parameter c decides whether to trade or not, upon being matched with a trader. Farmers trade whenever $p(c) > c$, they are indifferent between trading or not when $p(c) = c$ and we suppose that no farmer chooses to trade when $p(c) < 0$. In addition, whenever trade occurs, we impose the Nash Bargaining solution (5).[23] We assume that farmers only accept a price equal to or better than that from the Nash Bargaining solution.[24]

3. The value function equations (1)–(4) hold whenever there is (positive) flow in the BEM meaning that the participation constraint given by (2) and (4) hold for some c (i.e., the max occurs in the first term of the RHS of (2) and (4) for some values of c).

3.3 Proof of Existence of Equilibrium

Our method to prove the existence of equilibrium is to sequentially use equations (1)–(5) to prove the existence of $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$, $V^{FM}(c)$, V^{TU} which satisfy equations (1)–(5). We use equations (1)–(3) and (5) iteratively to eliminate $p(c)$ and the value functions $V^{TM}(c)$, $V^{FU}(c)$ and $V^{FM}(c)$ in equation (4) so that equation (4) becomes a function only of V^{TU}. We then show that this final version of equation (4) admits a fixed point or solution, V^{TU*}. The equilibrium value for V^{TU} then generates the equilibrium values of $p(c)$, $V^{TM}(c)$, $V^{FU}(c)$ and $V^{TU}(c)$ by re-tracing the initial substitutions. Here, we describe only the main steps of the proof, relegating details of intermediate steps to Appendix Section C.

[23] This is in keeping with the literature, where, as in this paper, the microfoundations of the Nash Bargaining rule process are not explicitly described.

[24] For example the trader could offer a take it or leave it offer to the farmer for a very small amount over the farmer's cost c, and the farmer could be modeled as being forced to take that offer. Instead we think of some within period match-specific game which justifies the Nash Bargaining posited here.

Making V^{TU} a function of parameters. In what follows, we define the following parameters to ease our exposition:

$$\sigma_1 = \phi(1-\delta) - \delta(1-\beta) + \beta\delta\phi + \delta\mu^F\phi + 1;$$

$$\sigma_2 = \sigma_1 - \mu^{\mathrm{F}}\phi.$$

Claim 1. σ_1 and σ_2 are positive.

Proof. In the appendix. □

We start by using equations (1) and (3) to obtain expressions for $V^{FU}(c)$ and $V^{FM}(c)$ as a function of $p(c)$:

$$V^{FM}(c) = \frac{(p(c)-c)\left(1-\delta+\delta\mu^F\right)}{(1-\delta)(1-\delta+\beta\delta+\delta\mu^F)}, \tag{6}$$

and

$$V^{FU}(c) = \frac{(p(c)-c)\,\delta\mu^F}{(1-\delta)(1-\delta+\beta\delta+\delta\mu^F)}. \tag{7}$$

Using the expressions above to eliminate $V^{FM}(c)$ and $V^{FU}(c)$ from equation (5), we obtain an expression for $p(c)$ as a function of c, $V^{TM}(c)$ and V^{TU}:

$$p(c) = c + \phi\left(1-\delta+\beta\delta+\delta\mu^F\right)(V^{TM}(c) - V^{TU}). \tag{8}$$

We use the equation above to eliminate $p(c)$ from $V^{FU}(c)$ in (7):

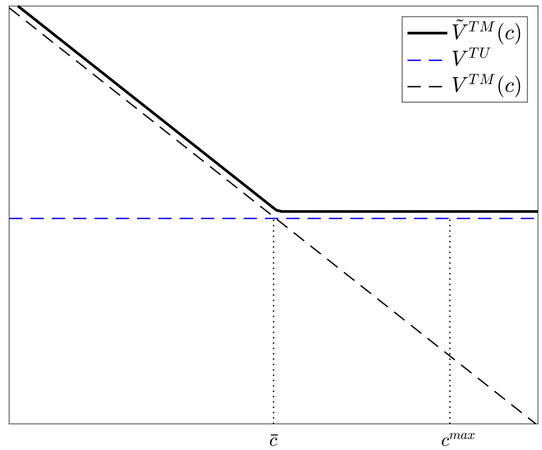
$$V^{FU}(c) = \frac{\delta\mu^F\phi\left(V^{TM}(c) - V^{TU}\right)}{1-\delta}. \tag{9}$$

We now move to expression (2). Our goal is to substitute the equations that we constructed so far into equation (2) to remove $p(c)$ and obtain an expression of $V^{TM}(c)$ as a function of c and V^{TU}. To do so, we construct an expression for $V^{TM}(c)$ that will be a solution to the fixed point problem defined in expression (2) when the left hand side of the maximum operator is larger than the right hand side, i.e., when the value for a trader of being matched to a farmer c is higher than the value of being unmatched.

First, let $V^{TM}_{RHS}(c)$ be the first term of the right hand side of equation (2): $V^{TM}_{RHS}(c) \equiv P - p(c) + \delta((1-\beta)V^{TM}(c) + \beta V^{TU})$. Replace $p(c)$ in this expression with equation (8) to get:

$$V^{TM}_{RHS}(c) = P - c - V^{TM}(c)(\sigma_1 - 1) + V^{TU}(\sigma_1 + \delta - 1). \tag{10}$$

Figure 2: The value function of matched traders



We will now compute the equation that we obtain when the left-hand side of equation (2) equals the first term of the maximum operator on the right hand side of the equation, i.e., when we have $V^{TM}(c) = V^{TM}_{RHS}(c)$. Isolate $V^{TM}(c)$ in this expression and define its solution (or fixed point) to be $\tilde{V}^{TM}(c)$:

$$\tilde{V}^{TM}(c) = \frac{\sigma_1 + \delta - 1}{\sigma_1} V^{TU} + \frac{P - c}{\sigma_1}. \tag{11}$$

Note that $\tilde{V}^{TM}(c)$ is linear and decreasing with respect to c with a slope of $-1/\sigma_1$. Figure (2) illustrates the shape $\tilde{V}^{TM}(c)$. Using this definition for $\tilde{V}^{TM}(c)$, the claim below defines an expression for $V^{TM}(c)$ as a function of V^{TU} and c.

Claim 2. Fix a V^{TU} and all the other parameters of the model, including c. Then $V^{TM}(c)$ is a solution to equation (2) if and only if $V^{TM}(c)$ is given by

$$V^{TM}(c) = \max\left\{\tilde{V}^{TM}(c), V^{TU}\right\}. \tag{12}$$

Proof. In the appendix. □

Let $\bar{c}$ denote the value of c when $\tilde{V}^{TM}(c)$ equals V^{TU}:

$$\bar{c} = P - (1 - \delta) V^{TU}. \tag{13}$$

The above tells us that V^{TU} needs to be less than $P/(1-\delta)$ otherwise $\bar{c}$ is negative. Also, if $V^{TU} = 0$ then $\bar{c} = P$. Figure 2 plots $V^{TM}(c)$ and $\bar{c}$. It shows that the total discounted utility value of being matched for a trader drops with farmers' cost c. This utility value drops down to the point $\bar{c}$ in which traders are indifferent between being matched with a farmer and being unmatched. The value V^{TU} thus sets a lower bound to the utility $V^{TM}(c)$ that traders obtain in trading partnerships with farmers.

Now replace $V^{TM}(c)$ in the expression for $V^{FU}(c)$ in expression (9) with the expression for $V^{TM}(c)$ obtained from equations (12) and (11) to obtain:

$$V^{FU}(c) = \frac{\delta\mu^F \phi\left(P - c - (1-\delta)V^{TU}\right)}{(1-\delta)\,\sigma_1} \text{ for } c < \bar{c} \tag{14}$$

and $V^{FU}(c) = 0$ for $c \geq \bar{c}$.

Note that $V^{FU}(c)$ in equation (14) is a function only of V^{TU} and the parameters of the model. We now have expressions for all of our equilibrium variables $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$ as functions of V^{TU}. By construction these expressions satisfy equations (1)–(5) except for (4). Hence, once we find an equilibrium value of V^{TU} satisfying equation (4), we can plug that into the just mentioned functions to generate equilibrium values of $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$. This is our next step. All expressions from now until the end of the proof will be functions of V^{TU} and the parameters of the model and otherwise independent of $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$. To find the final expression of V^{TU} as a function of the exogenous parameters of the model, we have to solve a fixed point problem which we define below.

Defining and solving the fixed point problem for V^{TU}. To define the fixed point problem that we use to find the solution for V^{TU}, let us work with expression (4). First, define

$$IV^{TM} = \int_0^{\bar{c}} \tilde{V}^{TM}(c) g(c) dc + \int_{\bar{c}}^{c^{max}} V^{TU} g(c) dc. \tag{15}$$

Next, define the first term on the right-hand side of expression (4) to be V^{TU}_{RHS}:

$$V^{TU}_{RHS} = \delta(\mu^T IV^{TM} + (1-\mu^T)V^{TU}) - \kappa. \tag{16}$$

It is easy to check that V^{TU}_{RHS} is a quadratic function of V^{TU} by inserting the equation we constructed for $\tilde{V}^{TM}(c)$ into IV^{TM} and integrating over the relevant values of c. Let a_{quad} be the coefficient of the quadratic term and a_{const} be the constant term (i.e., the value of V^{TU}_{RHS} when $V^{TU} = 0$). Simple but tedious algebra shows:

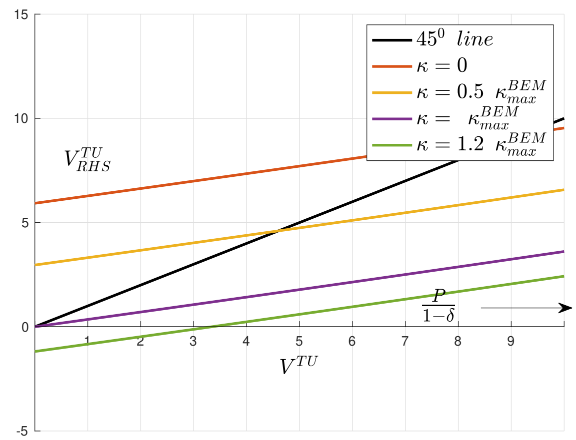
$$a_{quad} = \frac{\delta\mu^T (1-\delta)^2}{2c^{max}\sigma_1} \tag{17}$$

and

$$a_{const} = \kappa^{BEM}_{max} - \kappa \tag{18}$$

where

Figure 3: The fixed point problem of V^{TU} for different values of k in a BEM



Notes: This figure illustrates the fixed point problem that we solve. We constructed this figure based on the following parameter set [c^{max}, P, β, δ, μ^F, μ^T, ϕ] = [40, 40, 0.25, 0.5, 0.2, 0.8, 1].

$$\kappa_{max}^{BEM} = \delta \mu^T \frac{P^2}{2c^{max}\sigma_1}. \tag{19}$$

Since $\sigma_1 > 0$, then $\kappa_{max}^{BEM} > 0$. As we shall see below, κ_{max}^{BEM} represents the maximum value of κ such that we have an equilibrium with positive trade flows.

The claim that we present next proves some properties of expression V_{RHS}^{TU} that we use in our solution. The intuition behind the properties that we show is illustrated in Figure (3).[25] Figure (3) also highlights the fixed point problem that we have: the point at which V_{RHS}^{TU} crosses the 45 degree line gives the equilibrium value of V^{TU}. Note that κ_{max}^{BEM} is the maximum value of κ such that V_{RHS}^{TU} crosses V^{TU}.

Claim 3. The function V_{RHS}^{TU} has the following properties

1. V_{RHS}^{TU} is convex in V^{TU};

2. if $\kappa < \kappa_{max}^{BEM}$, then $V^{TU} - V_{RHS}^{TU} > 0$ at $V^{TU} = 0$, and, if $\kappa > \kappa_{max}^{BEM}$, then $V^{TU} - V_{RHS}^{TU} < 0$ at $V^{TU} = 0$;

3. at $V^{TU} = \frac{P}{1-\delta}$, $V^{TU} - V_{RHS}^{TU} = P + k > 0$.

[25] While our goal is not to provide a quantitative evaluation of the CEM, the parameter values that we pick for our figures are inspired by our institutional setting. We set the price of produce to 40 throughout, which is a common value in GHS of a bag of corn of 50 kg during the 2015-2018 period. We assume that $c^{max} = P$ so that, in principle, all farmers in a catchment area can generate positive market surplus if they sell their produce to traders. We use $\delta = 1/1.05$, which is a typical value in the literature and assumes a interest rate of 5% for the economy. We set μ^F to 0.2 and μ^T to 0.8 to capture the fact that there is an abundance of farmers per trader. Since we do not have a strong prior to the bargaining parameter ϕ, we set this value to 1, which gives an qual share of the gains from trade to traders and farmers. Given that there is recurrent new matches and broken matches according to our interviews, we set $\beta = 0.5$. In some figures, we deviate from these values to make our argument more salient. In Appendix Section D, we provide a more detailed discussion of these parameters based on comparative statics of the model.

Proof. (1) follows from the fact that $a_{quad} > 0$. (2) when $V^{TU} = 0$, $V^{TU}_{RHS} = a_{const}$ so part (2) follows immediately from equation (19). (3) Set $V^{TU} = P/(1-\delta)$ in V^{TU}_{RHS} and simple algebra proves part (c). □

Proposition 1. *For all $\kappa < \kappa^{BEM}_{max}$ there exists a unique equilibrium to the BEM model. When $\kappa > \kappa^{BEM}_{max}$ the equilibrium involves no trader making visits to farms.*

Proof. See details in the Appendix. □

Strategic Rejection. The last proposition concludes our proof. It shows that there exists an equilibrium with positive trade flows between farmers as long as the fixed costs for searching is not excessively high ($\kappa < \kappa^{BEM}_{max}$). In this equilibrium, there is a range of farmers with costs above $\bar{c}$ and below P who would generate positive gains from trade in the short-run, but who traders nevertheless reject in order to wait for the opportunity of being matched with farmers with lower costs. We call this phenomenon "strategic rejection." The next section discusses the aggregate supply that emerges in this steady state and the distributional gains from trade.

3.4 Gains from Trade and Aggregate Supply in a pure BEM

We now describe the aggregate supply and the distributional gains from trade in BEM. In the case of aggregate supply, we will have to track the matched and the unmatched farmers and determine their mass, which we left aside in the presentation of the model so far.

Gains from trade in a pure BEM. Figure 4a shows the price function and how the gains from trade are distributed between farmers and traders in the pure BEM.[26] It shows that farmers with costs above $\bar{c}$ and below P are rejected by traders, which generates a loss of potential matches equal to the area denoted by G relative to the optimum without search frictions. All farmers below $\bar{c}$ sell their produce at the bargained price $p(c)$. Area A represents the profits obtained by farmers when matched and area B the surplus or profits of the trader.[27]

[26] In the figure, farmers with cost $c > \bar{c}$ receive an offer of $p(c) = c$ from traders. To see how this is an equilibrium according to our definition, notice that traders do not have incentives to offer a price $p(c) > p$, since then farmers would accept their offer and traders prefer to reject any trade with such farmers, nor a price $p(c) < c$, since then traders would make negative profits. In addition, for $p(c) = c$, farmers are indifferent between accepting traders' offers or not and, for simplicity, we assume that they prefer not to sell their produce.

[27] In our analysis, the bargained price, $p(c)$, is increasing in the cost parameter c. One may object by saying that in the developing country context those with higher values of c should have weaker bargaining positions and therefore lower prices. In the Appendix, we explain why one should not expect this to be the case. We also point out here that the "profits" of farmers, that is, the difference between price and cost will most likely be decreasing in c.

Tracking matched and unmatched farmers in steady state. Fix a period t. First, note that the probability of being matched or unmatched is independent of the cost parameter c. So, we fix any c and let m_t be the probability that particular farmer of type c is matched. Alternatively, we can think of there being a unit mass of farmers with a given cost parameter c and m_t as being the fraction of them who are matched. Let u_t denote analogously the mass of farmers of a given cost parameter c who are unmatched in period t. In particular, $m_t + u_t = 1$. Since in each period the matched become unmatched with probability β and the unmatched become matched with probability μ^F, the mass of matched farmers of any given cost parameter c in period $t+1$ is given by

$$m_{t+1} = \mu^F u_t + (1-\beta)m_t. \tag{20}$$

and the mass of unmatched farmers is given by

$$u_{t+1} = (1-\mu^F)u_t + \beta m_t. \tag{21}$$

We look for steady state values m^* and u^* of m and u, defined to be the situation where $m_{t+1} = m_t = m^*$ and $u_{t+1} = u_t = u^*$. Simple algebra shows that the steady state values of m and u to be

$$u^* = \frac{\beta}{\beta + \mu^F} \tag{22}$$

and

$$m^* = \frac{\mu^F}{\beta + \mu^F}. \tag{23}$$

In steady state, each farmer type c will have a fraction m^* of its mass matched (or will have probability m^* of being matched) and a fraction $u^* = 1 - m^*$ unmatched.

In the earlier sections we ignored the distinction between the matched and unmatched and instead assumed there was some density function $g(c)$ of the relevant farmers which both traders and farmers use in computing their value functions (1)-(5). The relevant farmers used in those value function computations are those in the set of unmatched farmers. Suppose that the economy wide density function for all farmers, matched or unmatched, of different values of c is uniform on $[0, c^{max}]$, whose density function is $\frac{1}{c^{max}}$. Then the density function representing the unmatched farmers is

$$g(c) = \frac{u^*}{c^{max}}. \tag{24}$$

with u^* given by equation (22).[28] We shall often use this formulation in computing the supply

[28]For expositional convenience, we ignored the set of parameters $c > \bar{c}$, the strategically rejected farmers.

function, gains from trade and in comparing welfare in the BEM model just described with the dual economy with commodity exchange markets to be introduced later.

Aggregate supply in a pure BEM. We now characterize the aggregate supply of the economy in steady state. The active farmers in the economy are those with cost parameters between 0 and $\bar{c}(P)$. Only matched farmers will produce output in a given period, and, as argued above, the mass of those active farmers is m^*. Since each farmer produces one unit of output, the mass of the matched farmers is equal to the mass of output produced in the economy. The trade costs of farmers, whether matched or not, are uniformly distributed on $c \in [0, c^{max}]$. The aggregate supply of the economy $Q^{BEM}(P)$ is therefore

$$\begin{aligned} Q^{BEM}(P) &= \left(\int_0^{\bar{c}(P)} \frac{m^*}{c^{max}} dc \right) \\ &= \frac{\bar{c}(P)}{c^{max}} \frac{\mu^F}{\beta + \mu^F}. \end{aligned} \tag{25}$$

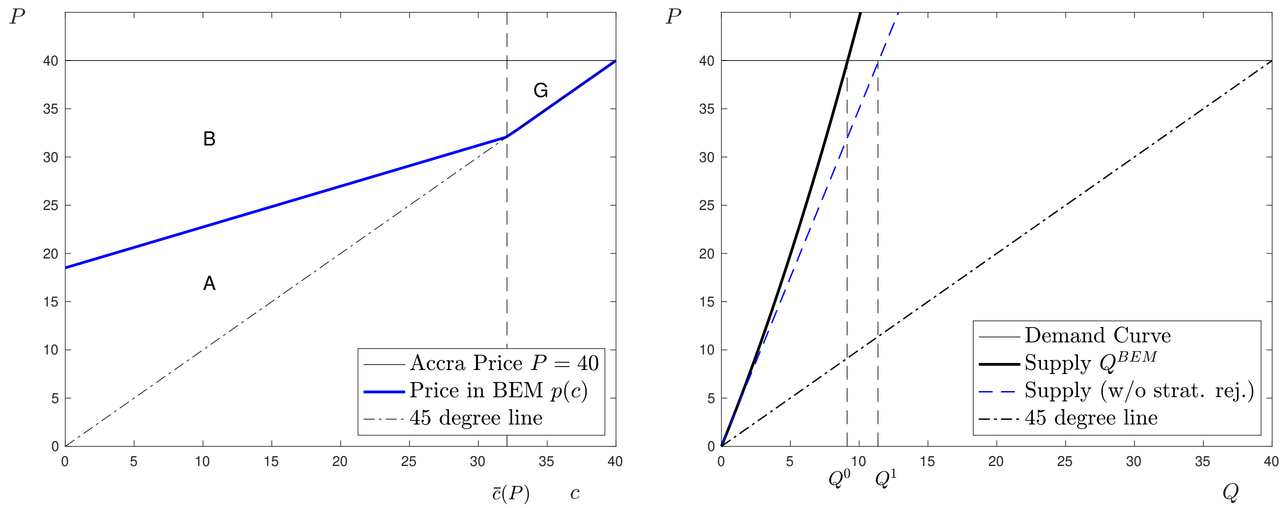
Figure (4b) draws familiar supply and demand figures based on our model. The demand curve is represented by the horizontal black line (for expositional convenience we assume to be perfectly elastic, which has some justification as the country is small with imports of grains internationally and from the 3 countries it shares land borders with; it is of course easy to draw a downward sloping demand curve in its place). The supply curve is given in equation (25) and is represented by the solid black curve.[29] In our model, the types of farmers that are matched and so send produce to the market are those in the set $[0, \bar{c}(P)]$. Hence, if we did not have to worry about the matches and random separations of traders and farmers, the total output at any price P would be $c(P)$. This can be seen in equation (25) when $\mu^F / \left(\beta + \mu^F\right) = 1$. To highlight the role of strategic rejection, we also add to the figure the supply curve for the situation where trader reach an agreement negotiation with any farmer whose cost c is below P—that is, if there is no strategic rejection. In that case, all farmers in the set $[0, P]$ would generate a market surplus and send their produce to the market when they are matched to a trader. We also include one other potential supply curve. This is the conventional supply curve without any search and without the destruction of matches. This of course would be the 45 degree line, which is also drawn in the figure.

Equations (20) - (23) hold for values of c outside of this set. For values of c in this set, the density function remains $g(c)$ but those values of c produce no output. We account for this in the supply function below.

[29] Note that we set $\kappa = 0$ to ensure that there are always mass of farmers selling their produce to traders for any $P > 0$. With $\kappa > 0$, we have a discontinuity in the supply function at $c = \kappa$ since for low enough prices we have no trade between farmers and traders.

Figure 4: Supply of Agricultural Goods in a Bilateral Exchange Market

(a) Prices and Distribution of Gains, $p(c)$ (b) Aggregate Supply, Q^{BEM}



Notes: Panel (a) shows the distribution of prices obtained by farmers with different cost parameter c. Panel (b) shows the aggregate supply of agricultural produce. In Panel (b), $Q^0 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve that excludes the quantity produced by strategically rejected farmers crosses the demand curve at $P = 40$ and $Q^1 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{P}{c^{max}}$ is the point where the aggregate supply that adds the strategically rejected farmers crosses the demand curve at that same retail price. The 45 degree line in Panel (b) represents the conventional supply curve. We constructed this figure using [c^{max}, P, β, δ, κ, μ^F, μ^T, ϕ]= [40, 40, 0.5, 0.952, 0, 0.2, 0.8, 1].

4 The Dual Market Economy with the Commodity Exchange Market (CEM)

This section introduces a commodity exchange market (CEM) into the economy. We provided a description of the commodity exchange and how it functions earlier in the paper. As we discussed earlier, the commodity exchange has some brokers who receive produce from the farmers and other brokers who sell the produce to final buyers at a price P. Farmers can bring their produce and sell it to the members of the commodity exchange if they pay a transaction cost of τ. This transaction cost captures the operational costs that the commodity exchange incurs with warehouses and the brokers. The commodity exchange has an auctioneer whose task is to equalize the demand from its members with the supply of farmers who brought their produce. This auctioneer operates as a Walrasian auctioneer. The price at the commodity exchange is the final price P, what we referred to earlier as the Accra price.

4.1 Value function of farmers in the CEM

Analogous to how we modeled unmatched farmers in the BEM, unmatched farmers who choose to sell to the commodity exchange have to wait a period, thereafter they can receive a value of $P(1-\tau)-c$ in each period.[30] The total discounted return of selling their produce to the commodity exchange is therefore:‘

$$V^{FCE}(c) = \frac{\delta}{1-\delta}(P(1-\tau)-c). \tag{26}$$

4.2 Definition of Equilibrium in Dual Market

Definition 2. An equilibrium for this model with the given parameter set $\Omega = \{c^{max}, P, c, \beta, \delta, \kappa, \mu^F, \mu^T, \phi, \tau\}$ is a BEM equilibrium as in the earlier definition for a set of c's (which may be empty or the set of all c's) such that on the set where the BEM holds, conditions (1)–(5) with (26) replacing (4) hold and we add a new condition (6): Each farmer of cost parameter c decides whether to go to the CEM or BEM depending upon whether $V^{FCE}(c)$ is bigger than $V^{FU}(c)$ or not.

4.3 Proof of the Existence of Equilibrium in Dual Markets

To start, fix any value of V^{TU} in the BEM, and for any farmer with cost parameter c, perform all the substitutions in equations (6)–(14) so that we have all our potential equilibrium variables $V^{FU}(c)$, $V^{FM}(c)$, $V^{TM}(c)$ and $p(c)$ stated in terms of V^{TU}. These satisfy all of the required equations (1)–(5) except for equation (4). For any farmer type c, if that farmer goes into the BEM, those equations will hold for that c. This procedure is the same that we used in the proof for the existence of an equilibrium in the BEM model. The difference here is that here we also need to determine, for each farmer of type c, whether she goes to the CEM or the BEM, which we do next.

The set of farmers who sell to the BEM will be determined in equilibrium and be a function of V^{TU}. In comparison to the proof in the BEM case, with the existence of a CEM the expressions for IV^{TM} in equation (6) and V^{TU}_{RHS} will change. Specifically, the set of c's in the integrals in IV^{TM} will equal those who decide to go to the BEM.

Our method to prove the existence of an equilibrium in the dual markets model will be to fix a V^{TU}, find the set of farmers who choose to sell in the BEM, adjust the integrals in IV^{TM} in equation (6) and find a fixed point value for V^{TU} for equation (4) with the revised IV^{TM}.

[30] The timing per se is not too relevant, it would scale up or down the value functions by a fixed constant factor depending on the discount factor, but we do need to stick one timing scheme and keep it consistent between the BEM and CEM for the valid comparisons of the different regimes. (The scaling factor if all trades take place in the same period would be $\frac{\delta}{1-\delta}$,or its reciprocal.)

Once we find an equilibrium value of V^{TU} satisfying equation (4), we can plug that back into value functions (6)–(14) to generate an equilibrium in the BEM part of the model.

We next provide a set of claims associated with farmers' choice to sell to the CEM or to the BEM. We will determine the set of farmers going to each market as a function of V^{TU}. We show that, in an equilibrium with positive trade flows in CEM, there is a partition of c such that farmers with c below some value $\underline{c}$ sell to the CEM and those with c between $\underline{c}$ and $\bar{c}$ stay in the BEM. (Recall that the farmers with a trade cost c above $\bar{c}$ are those who are strategically rejected.) Once we establish this partition and how it depends on V^{TU}, we use our results to search for values of V^{TU} that characterize a solution for equation (4).

Farmers' choice between the BEM and CEM and the partition of c. From equation (14), the value function $V^{FU}(c)$ of the farmer in the BEM is decreasing and linear in c (for $c < \bar{c}$) with intercept and slope given respectively by

$$V_{const}^{FU} = \frac{\delta \mu^F \phi \left(P - (1-\delta)\mathrm{V}^{\mathrm{TU}}\right)}{(1-\delta)\,\sigma_1} \tag{27}$$

and

$$V_{slope}^{FU} = -\frac{\delta \mu^F \phi}{(1-\delta)\,\sigma_1}.$$

For $c \geq \bar{c}$, $V^{FU}(c)$ is zero. Similarly, from equation (26) the value function $V^{FCE}(c)$ of the farmer in the CEM is decreasing and linear in c with slope and intercept given by

$$V_{slope}^{FCE} = \frac{\delta}{\delta - 1} \tag{28}$$

$$V_{const}^{FCE} = \frac{P\delta\,(1-\tau)}{1-\delta}. \tag{29}$$

Claim 4. The slope of $V^{FCE}(c)$ is steeper than that of $V^{FU}(c)$: i.e., abs(V_{slope}^{FCE}) > abs(V_{slope}^{FU}).

Proof. In the appendix. □

The claim above shows that, from a farmers' perspective, the value of being unmatched in the BEM does not fall as much with the cost parameter c as the value of selling their produce to the CEM. This occurs because, as c rises, the present value of all the reductions in the future streams of sales must be discounted by the fact that farmers do not sell any produce in future periods in which they are unmatched. This differential effect of c on $V^{TU}(c)$ and $V^{FCE}(c)$ gives rise to an intersection point which we define by $\underline{c}$. Simple algebra shows this to be:

$$\underline{c} = P\left(\frac{\tau_2 - \tau}{\sigma_1 \sigma_2}\right) + \frac{\mu^F \phi (1-\delta)}{\sigma_2} V^{TU} \tag{30}$$

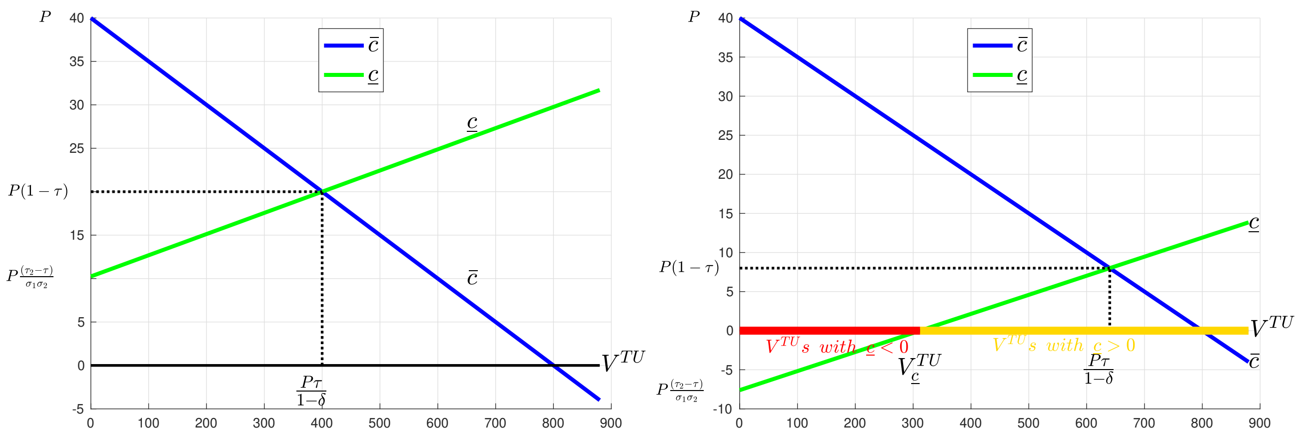
where

$$\tau_2 \equiv \frac{\sigma_2}{\sigma_1}. \tag{31}$$

The value of $\underline{c}$ will determine, in the cases in which the CEM and BEM coexist, the cutoff value below which farmers choose to sell to the CEM. As in the pure BEM case, we also define a value $\bar{c}$ given by equation (13), above which farmers who are strategically rejected by traders. Note that these (i.e., $\underline{c}$ and $\bar{c}$) can be considered functions of V^{TU} (holding fixed the parameter values).

Figure 5: $\bar{c}$ and $\underline{c}$ as functions of V^{TU}

(a) if $\underline{c} > 0$ for all $V^{TU} \geq 0$ (b) if $\underline{c} \leq 0$ for all $V^{TU} \in [0, V_{\underline{c}}^{TU}]$



Notes: Panel (a) uses parameter values [c^{max}, P, β, δ, κ, μ^F, μ^T, ϕ, τ] = [40, 40, 0.5, 0.952, 0, 0.5, 0.5, 1, 0.5], which ensures that $\tau < \tau_2$. Panel (b) 5b uses the same parameter values except that it changes τ from 0.5 to 0.8, which ensures that $\tau > \tau_2$.

In what follows, we define

$$V_{\underline{c}}^{TU} = -P\left(\frac{\tau_2 - \tau}{\mu^F \phi (1-\delta) \sigma_1}\right), \tag{32}$$

which is the value of V^{TU} when $\underline{c} = 0$ in expression (30). For any value of V^{TU} below $V_{\underline{c}}^{TU}$, no farmer will choose to sell to the CEM. Therefore, $V_{\underline{c}}^{TU}$ will be the lowest value of V^{TU} that is consistent with existence of the CEM. In Figure (5a), $\tau < \tau_2$ so $V_{\underline{c}}^{TU} < 0$ and $\underline{c} > 0$ for all $V^{TU} \geq 0$. In Figure (5b), $\tau > \tau_2$ so $V_{\underline{c}}^{TU} > 0$ and, as Claim (5) below states, $\underline{c}$ is negative or positive depending upon whether V^{TU} is smaller or bigger than $V_{\underline{c}}^{TU}$.

Claim 5. $\bar{c}$ and $\underline{c}$ are functions of V^{TU} and have the following properties:

1. $\bar{c}$ is linear and decreasing in V^{TU} with value P at $V^{TU} = 0$;

2. $\underline{c}$ is linear and increasing in V^{TU} with value $P(\frac{\tau_2-\tau}{\sigma_1\sigma_2})$ at $V^{TU}=0$;

3. If $\tau < \tau_2$, then $\underline{c} > 0$ for all V^{TU} and, if $\tau > \tau_2$, then $\underline{c}$ is negative for V^{TU} in $[0,\, V_{\underline{c}}^{TU})$ and positive for V^{TU} in $(V_{\underline{c}}^{TU},\, \infty)$;

4. $\underline{c}$ and $\bar{c}$, as functions of V^{TU}, have a common point of intersection at $V^{TU} = P\tau/(1-\delta)$, where they equal $P(1-\tau)$.

Proof. (1) - (4) follow from the definitions in (13) and (30)–(32). □

Figure 6 illustrates the results from Claim (5). There are three partitions of c depending upon the values of $\bar{c}$ and $\underline{c}$ in relation to each other and to whether $\underline{c}$ is positive or negative. One can think of Figure (6) as follows. Think of the $V^{FCE}(c)$ function (in yellow) as fixed. Then the $V^{FU}(c)$ function, which we know is flatter than the $V^{FCE}(c)$ function (in red), can be in one of three positions relative to the V^{FU} function: it can intersect it (Case I), it can lie above it (Case II) or lie below it (Case III). Each of these positions is associated with a different configuration of farmers in terms of their choices of selling to the BEM or the CEM.

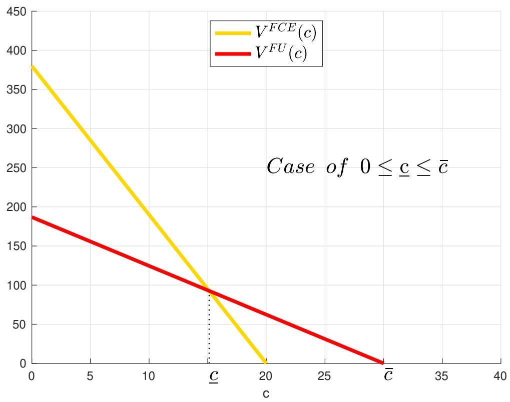
Case I ($0 \le \underline{c} \le \bar{c}$) **(Dual Markets).** In Case I, we have a region $[0,\underline{c})$ of c's where the CEM dominates the BEM and another interval $(\underline{c},\bar{c})$ where the BEM dominates the CEM. Case I is a Dual Market economy. Here, the $V^{FU}(c)$ function intersects the $V^{FCE}(c)$ function, and this intersection occurs when either $V_{\underline{c}}^{TU}$ is negative and V^{TU} is in $(0,\, \frac{P\tau}{1-\delta})$ or $V_{\underline{c}}^{TU}$ is positive and V^{TU} is in $(V_{\underline{c}}^{TU},\, \frac{P\tau}{1-\delta})$.

Case II ($\underline{\mathbf{c}} < 0 \le \bar{c}$) **(Pure BEM).** This is the case where the $V^{FU}(c)$ function lies above the $V^{FCE}(c)$ function, and occurs when $V_{\underline{c}}^{TU} > 0$ and V^{TU} is in the set $[0, V_{\underline{c}}^{TU})$. In Case II the BEM dominates the CEM for all relevant values of $c > 0$. Case II is a BEM only economy.

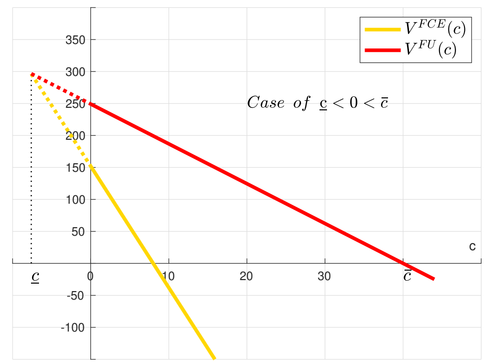
Case III ($\bar{c} < \underline{c}$) **(Pure CEM).** This is the case where the $V^{FU}(c)$ function lies below the $V^{FCE}(c)$ function, and occurs when $V^{TU} > \frac{P\tau}{1-\delta}$. In Case III, the CEM dominates the BEM for all relevant values of $c > 0$. This case will not feature in the equilibria we will describe later. The reason is that in this case the CEM dominates the BEM so there would be no BEM and the concept of a V^{TU} would not make sense as V^{TU} pertains to the the BEM model which would not even exist.

Figure 6: Relationship between $\bar{c}$ and $\underline{c}$

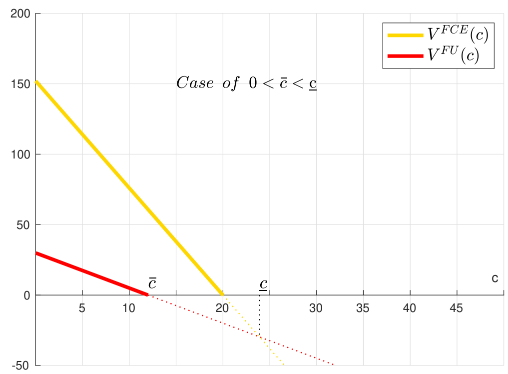
(a) Case I: $0 < \underline{c} < \bar{c}$



(b) Case II: $\underline{c} < 0 < \bar{c}$



(c) Case III: $0 < \bar{c} < \underline{c}$



In this sub-section, we defined farmers choice between selling to the CEM and the BEM in terms of three types of partitions of c. In the next section, we turn to define the fixed point problem for V^{TU}, which we use to find the equilibrium value of V^{TU} as a function of the exogenous parameters of the model.

Defining and solving the fixed point problem for V^{TU}. To define the fixed point problem that we use to establish the equilibrium, we work with expression (4) (as we did to prove the equilibrium in BEM). The limits of integration in the integral in equation (4) should be over the set of c's which go to the BEM. That set will differ depending upon whether we are in one of the three types of partitions of c as defined by Case I, II or III described above. We shall define first the integral that would be the correct integral if we were in Case I and define the expression as IV^{TM}_{DUAL}:

$$IV^{TM}_{DUAL} = \int_0^{\underline{c}} V^{TU} g(c)dc + \int_{\underline{c}}^{\bar{c}} \tilde{V}^{TM}(c)g(c)dc + \int_{\bar{c}}^{c^{max}} V^{TU} g(c)dc \tag{33}$$

where, recall, $g(c) = 1/c^{max}$ is the uniform density on $[0,\ c^{max}]$, and

$$V^{TU}_{RHS,DUAL} = \delta(\mu^T I V^{TM}_{DUAL} + (1-\mu^T)V^{TU}) - \kappa. \tag{34}$$

Using the expression for $\tilde{V}^{TM}(c)$, it is easy to verify that the expression for $V^{TU}_{RHS,DUAL}$ above is a quadratic function of V^{TU}. Let $a_{quad,DUAL}$ be the coefficient of the quadratic term of $V^{TU}_{RHS,DUAL}$ and $a_{const,DUAL}$ be the constant term of this quadratic function, simple but tedious algebra gives:

$$a_{quad,DUAL} = \frac{\delta\mu^T(1-\delta)^2\sigma_1}{2c^{max}\sigma_2^2}, \tag{35}$$

and

$$a_{const,DUAL} = \kappa^{Dual}_{max} - \kappa, \tag{36}$$

where

$$\kappa^{DUAL}_{max} = \frac{\delta P^2\mu^T\tau^2\sigma_1}{2c^{max}\sigma_2^2}. \tag{37}$$

Analogous to Claim (3), it is easy to show using equations (35) and (36) that we have the properties of $V^{TU}_{RHS,DUAL}$ listed in the claim below.

Claim 6. The function $V^{TU}_{RHS,DUAL}$ has the following properties:

1. $V^{TU}_{RHS,DUAL}$ is convex in V^{TU};
2. if $\kappa < \kappa^{DUAL}_{max}$, then $V^{TU} - V^{TU}_{RHS,DUAL} < 0$ at $V^{TU} = 0$, and, if $\kappa > \kappa^{DUAL}_{max}$, then $V^{TU} - V^{TU}_{RHS,DUAL} > 0$ at $V^{TU} = 0$;
3. at $V^{TU} = \frac{P\tau}{1-\delta}$, $V^{TU} - V^{TU}_{RHS,DUAL} = P\tau + \kappa > 0$.
4. As a function of V^{TU}, the function $V^{TU}_{RHS,DUAL}$ has a unique fixed point on $[0, \frac{P\tau}{1-\delta})$.

The expression (34) would be the correct variant of the right hand side of (4) if Case I holds, that is, if farmers sell to both BEM and CEM. If Case II holds, since the BEM is chosen for all c, then expression (16) would be the correct variant of the right hand side of (4). When Case III holds the CEM is chosen at all values of c.

Different from our proof strategy in the case of a pure BEM, in the construction of the fixed point here we have to consider that the relevant variant of the right hand side of equation (4) will depend on each configuration of farmers who select into each type of market. For clarity, let us define $V^{TU}_{RHS,BEM}$ as the variant of V^{TU}_{RHS} which occurs when all farmers go to the BEM. By definition, at $V^{TU} = V^{TU}_{\underline{c}}$, we have $\underline{c} = 0$ and it is easy to check that $V^{TU}_{RHS,DUAL} = V^{TU}_{RHS,BEM}$,

Table 1: Types of Equilibria (assume small k)

Condition on parameters	Equilibrium
$V_{\underline{c}}^{TU} > 0$ and $\overline{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$ (point of intersection is positive) (intersection above 45deg. line)	Dual Economy with BEM and CEM See Figure 7
$V_{\underline{c}}^{TU} > 0$ and $\overline{V}_{\underline{c}}^{TU} < V_{\underline{c}}^{TU}$ (point of intersection is positive) (intersection below 45deg. line)	BEM only See Figure 7
$V_{\underline{c}}^{TU} < 0$ (point of intersection is negative)	Dual Economy with BEM and CEM

since the integrals defining them will be the same. Based on this observation, we define $\overline{V}_{\underline{c}}^{TU} \equiv V_{RHS,DUAL}^{TU} = V_{RHS,BEM}^{TU}$ when $V^{TU} = V_{\underline{c}}^{TU}$. With these definitions, we now have the elements that we need for our proof strategy.

Figures (7a) and (7b) will be critical in the explanation of our proof. The graphs each show $V_{\underline{c}}^{TU}$ on the horizontal axis as the value of V^{TU} where the functions $V_{RHS,BEM}^{TU}$ and $V_{RHS,DUAL}^{TU}$ intersect. They also show the common value of these functions at their point of intersection, $\overline{V}_{\underline{c}}^{TU}$. We will delineate a number of cases depending upon whether the point of intersection occurs above the 45 degree line (as in Figure (7a)) or below it (as in Figure (7b)).

Note that both $\overline{V}_{\underline{c}}^{TU}$ and $V_{\underline{c}}^{TU}$ are functions of the parameters of the model and are independent of V^{TU}. Proposition (2) below gives us conditions in terms of the parameters of the model where different types of equilibria (BEM and/or CEM) exist. We illustrate this in Table 1. Through numerical examples we will provide, we show that each of these cases is satisfied for some parameter values.[31]

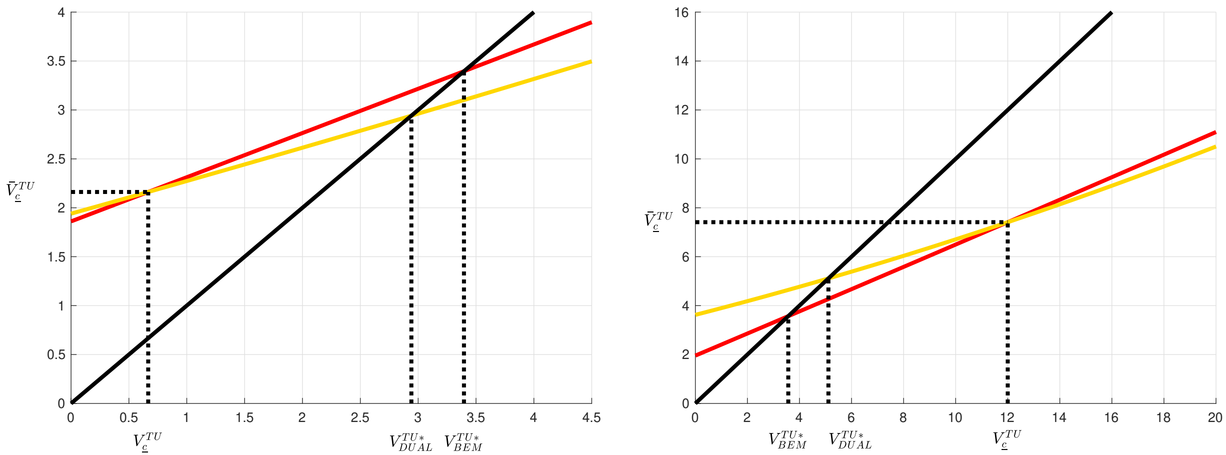
Proposition 2. *Suppose that* $\kappa < \min\{\kappa_{max}, \kappa_{max}^{DUAL}\}$*. (I) Suppose that either* $V_{\underline{c}}^{TU} > 0$ *and* $\overline{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$ *or* $V_{\underline{c}}^{TU} < 0$*. Then there is a unique equilibrium and that equilibrium is a dual economy. (II) Suppose that* $V_{\underline{c}}^{TU} > 0$ *and* $\overline{V}_{\underline{c}}^{TU} < V_{\underline{c}}^{TU}$*. Then there is a unique equilibrium and that equilibrium is a BEM only economy.*

Proof. Figure (7) provides intuition to our proof. In each panel, we have two lines: a yellow one that represents $V_{RHS,DUAL}^{TU}$ and a red one that represents $V_{RHS,BEM}^{TU}$. Each of these two lines represent a different variant of the right hand side (RHS) of equation (4). The point where they intersect each other gives $V_{\underline{c}}^{TU}$ in the horizontal axis and $\overline{V}_{\underline{c}}^{TU}$ in the vertical one. Figure

[31] These conditions on the parameter space are exhaustive in the sense that every any set of parameter values will necessarily fall into one of these regions. This is because we look at the parameters where $V_{\underline{c}}^{TU}$ is either positive or negative and for the former case we sub-divide into two other exhaustive cases ($\overline{V}_{\underline{c}}^{TU}$ bigger than or smaller than $V_{\underline{c}}^{TU}$). We ignore of course the knife-edge cases where the inequality is replaced with equality as it is easy to see what will happen in those cases—they will be the knife edge cases of the proposition.

Figure 7: Different cases for V_{BEM}^{TU*} and V_{Dual}^{TU*}

(a) Case $V_{\underline{c}}^{TU} > 0$ and $\bar{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$. (b) Case $V_{\underline{c}}^{TU} > 0$ and $\bar{V}_{\underline{c}}^{TU} < V_{\underline{c}}^{TU}$.



(7a) illustrates the case in which the two lines intersect each other at a point that is above the 45 degree line. In this case, we know that $\overline{V}_{\underline{c}}^{TU} > V_{\underline{c}}^{TU}$, which implies that the correct variant of the RHS of equation (4) is the yellow line, since some farmers choose to sell to the CEM and some to the BEM. Therefore, in Figure (7a) the solution to the of the system is given by V_{DUAL}^{TU*} instead of V_{BEM}^{TU*}. A similar logic can be applied to Figure (7b), in which the two lines intersect each other below the 45 degree line. In this second case, the correct variant of the RHS of equation (4) is $V_{RHS,BEM}^{TU}$ and we can rule out V_{DUAL}^{TU*} as a solution to the problem. Once we have identified the correct variant of the RHS of equation (4), we can then follow steps very similar to that used in proving the existence of equilibrium in the pure BEM economy (Proposition 1) above. We therefore obtain the existence of an equilibrium in the economy where both the CEM and BEM are possible. The conditions (I) and (II) of this proposition then gives us the situations where we have a dual or a BEM only economy. See additional details in the appendix. □

Proposition (2) above provides for us the existence of an equilibrium value of V^{TU}. As remarked earlier, this value of V^{TU}, in turn provides us with equilibrium values of all the other value functions as well as the pricing function $p(c)$, by retracing the steps we took to get V^{TU}. Figure (7a), which was drawn with parameter values $[c^{max}, P, \beta, \delta, \mu^F, \mu^T, \phi, \tau] = [40, 40, 0.1, 0.5, 0.1, 0.9, 0.5, 2, 0.3]$, shows the equilibrium value of V^{TU} to be V^{TU*} =2.94. Retracing our steps results in the following values of the different value functions: $\underline{c}^* = 2.73$, $\bar{c}^* = 38.53$, $V^{FU*} = 27.20 - 0.71c$, $\tilde{V}^{TM*}$ =$18.06 - 0.39c$, $p(c) = 30.21 + 0.22c$, and V^{FM*} =$57.42 - 1.49c$.

Figure (7b), drawn with parameter values $[c^{max}, P, \beta, \delta, \mu^F, \mu^T, \phi, \tau] = [40, 40, 0.1, 0.5, 0.005, 0.9, 0.5, 2, 0.4]$, shows the equilibrium value of V^{TU} to be V^{TU*} =3.57. Retracing our steps results in the following values of the various value functions and other functions: $V^{FU*} =$

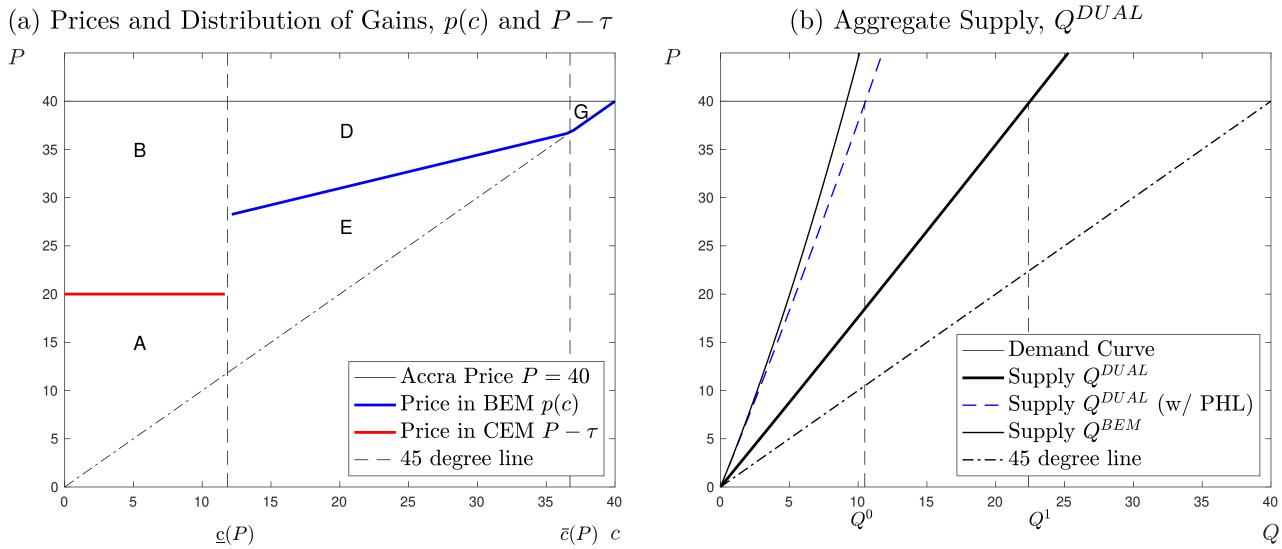
$26.98 - 0.71c$, $\tilde{V}^{TM*} = 18.56 - 0.39c$, $V^{FM*} = 56.95 - 1.49c$, $p(c) = 29.97 + 0.22c$, $\underline{c}^* = -10.12$, and $\bar{c}^* = 38.22$.

We remark in closing that when k is large, and indeed $\kappa > \max\{\kappa_{max}, \kappa_{max}^{DUAL}\}$ then arguments similar to those used in Proposition (1) indicate that there will not exist an equilibrium.

4.4 Aggregate Supply and Gains from Trade in Dual Market Economy

We now describe the characteristics of the equilibrium in the economy with the potential for dual markets. We will look at the distributional gains from just as we did earlier in the economy where only the BEM was allowed.

Figure 8: Aggregate Supply of Agricultural Goods in a Dual Market



Notes: Panel (a) shows the distribution of prices obtained by farmers with different cost parameter c. Panel (b) shows the aggregate supply of agricultural produce. In the figure, $Q^1 \equiv \frac{\beta}{\beta+\mu^F}\frac{\underline{c}}{c^{max}} + \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve in dual markets crosses the demand curve at $P = 40$ and $Q^0 \equiv \frac{\mu^F}{\beta+\mu^F}\frac{\bar{c}}{c^{max}}$ is the point where the aggregate supply curve in dual markets, excluding the post-harvest losses (PHL) avoided by farmers when they sell to the CEM, crosses the demand curve at that same price.

Gains from trade in dual markets. Figure (8a) shows the price function in the Dual Market Economy. The figure[32] highlights the gains from trade between farmers, traders and the commodity exchange. Area B represents the revenues generated by the fee of the CEM, area A represents the profits of farmers who sell to the commodity exchange, area D the surplus of traders in the BEM and area E the profits of farmers who sell to the BEM. Area G is a sort

[32]This graph is generated from the following parameters, chosen to make the graph and the different parts clear: [c^{max}, β, δ, k, μ^F, μ^T, ϕ, τ]= [40, 0.5, 0.952, 0, 0.2, 0.8, 1, 0.5].

of deadweight loss, representing the lost trading opportunities of farmers who are strategically rejected.

At $\underline{c}$, there is a gap between the price obtained by farmers in the CEM by farmers and the price obtained by farmers in the BEM. This gap captures the tradeoff between the certain price obtained by farmers in the CEM and the price volatility in the BEM. When farmers negotiate with traders, they incorporate in their utility value the fact that selling in the BEM comes with a probability of not being matched with any trader in future periods, i.e., it comes with the probability of incurring post-harvest losses. This occurs because there exists a risk in the search process that is internalized in the price of produce in the BEM.

Aggregate supply in dual markets. As we see from the analysis above, in the dual economy we have farmers with cost parameters in $[0,\underline{c})$ (when $\underline{c} > 0$) selling to the CEM and those with cost parameters in $(\underline{c},\bar{c})$ selling to the BEM. All farmers with cost parameters in $[0, \underline{c})$ are "matched" (to the CEM) with probability one. For farmers who sell to the BEM, we have a steady state probability m^* of being matched which is the same as the one that we derived in the pure BEM market (see equation (23)). Following our previous assumption, we assume that the set of all farmers—matched and unmatched —is, as before, uniform on $[0, c^{max}]$ and that there is a unit mass of farmers. The aggregate supply in the dual economy, $Q^{DUAL}(P)$, is thus given by[33]

$$\begin{aligned} Q^{DUAL}(P) &= \int_0^{\underline{c}(P)} (\frac{1}{c^{max}})dc + \int_{\underline{c}(P)}^{\bar{c}(P)} (\frac{m^*}{c^{max}})dc \\ &= \frac{\underline{c}(P)}{c^{max}} + \frac{(\bar{c}(P) - \underline{c}(P))}{c^{max}} \frac{\mu^F}{\beta + \mu^F} \\ &= \frac{\underline{c}(P)}{c^{max}} \frac{\beta}{\beta + \mu^F} + \frac{\bar{c}(P)}{c^{max}} \frac{\mu^F}{\beta + \mu^F} \end{aligned} \tag{38}$$

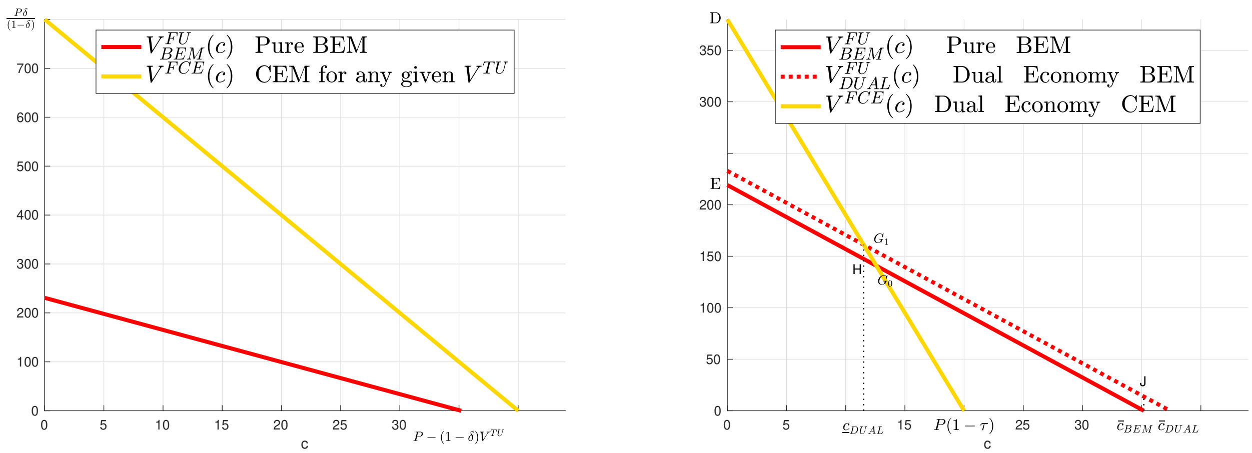
We illustrate our analysis with Figure (8b), which uses the same parameter values as in Figure (4). The thick black line shows the supply curve, $Q^{DUAL}(P)$, in the dual market economy. We have also shown, for the same parameter values, the supply curve in the economy where there is only one BEM market and the CEM is not allowed—that is the thin black line and is the same supply curve as in the BEM only economy of Figure (4b).

In dashed blue line we have the supply in the dual market economy all farmers below $\bar{c}$ sold their produce to the BEM and incurred the post harvest losses associated with not being matched with traders. In this case, only a share $\mu^F / \left(\beta + \mu^F\right)$ of all farmers would sell to BEM markets in steady state. The remainder are those who are unmatched. With the commodity

[33]This is for the case when $\underline{c} > 0$; when $\underline{c} \leq 0$ we replace $\underline{c}$ in the equation (30) with 0, in which case the formula collapses to the expression for the BEM only economy discussed earlier (see (25)).

Figure 9: Gains from introducing a CEM

(a) Without tax ($\tau = 0$) (b) Dual Economy



exchange, farmers with costs below $\underline{c}$ sell their produce to the CEM and are matched with probability one. The difference between the blue line and the thin black line is therefore the improvement in supply due to the role of the commodity exchange in changing the share of farmers with a match from the fraction $\mu^F/\left(\beta+\mu^F\right)$ to the fraction 1. (One may ask why the dotted blue line is not the same as the thin black line. After all, they are both situations where only the BEM operates. The reason is that they are two different equilibria with different values of $\bar{c}$.)

4.5 Welfare Gains from the Introduction of the Commodity Exchange Market

The earlier section looked at the effect on the supply and supply curve of the introduction of the possibility of the CEM. Let us now consider the welfare gains from introducing the CEM in the economy.

To start, consider the case where $\tau = 0$. Figure (9a) shows the welfare of unmatched farmers with different parameter cost c when $\tau = 0$. In this case, it is easy to see that the commodity exchange is unambiguously better than the BEM for all farmers.

With a positive tax and a dual economy, comparing the welfare requires some caution since the introduction of the CEM changes the operation of the BEM in the dual economy. In other words, the BEM itself will change when it goes from being the only market to being one of two markets together with the CEM.

In Figure (9b), we draw in yellow the value function of the farmer in the CEM, $V^{FCE}(c)$ from equation (26). We also draw the value function of the farmer in the BEM, $V^{FU}(c)$ in equation (14). There are two variants of this value function. We draw the value function in equilibrium in the situation where there is only a BEM (before the introduction of the CEM) and denote this as $V^{FU}_{BEM}(c)$. This value function is defined with some equilibrium value of V^{TU} equal to V^{TU*}_{BEM}. With the introduction of the CEM, there is a change in the equilibrium and a new value of V^{TU}, which we define as V^{TU*}_{DUAL} . We denote the farmer's value function in this case by $V^{FU}_{DUAL}(c)$. In Figure (7a) where we have a dual economy, note that the equilibrium value of $V^{TU*}_{DUAL} < V^{TU*}_{BEM}$. As easily seen from the formula for $V^{FU}(c)$ in equation (14), $V^{TU*}_{DUAL} < V^{TU*}_{BEM}$ implies that the value function $V^{FU}_{DUAL}(c)$ is always higher than that of $V^{FU}_{BEM}(c)$ (i.e., for all c).

The results from Figure (9b) show that the BEM in the pure BEM economy has a lower value for unmatched farmers (the thick red line) than the BEM in a dual economy (the dotted red line). The reason for this is that the equilibrium value for unmatched traders, V^{TU*}, is higher in the pure BEM model in comparison with the dual economy. Unmatched traders do better in the pure BEM economy than in the dual market economy. This occurs because farmers with the lowest costs parameter c prefer to sell to the CEM, and these farmers were the ones with whom traders were obtaining the largest gains from trade. Since these farmers are not available to traders anymore, they need to settle for lower values in the dual economy. As a consequence, they also lower their standards for strategic rejection, and start accepting farmers with higher costs c, which is shown in the figure by a move from $\bar{c}_{BEM}$ to $\bar{c}_{DUAL}$. This can be easily seen in equation (13) when $V^{TU*}_{DUAL} < V^{TU*}_{BEM}$. We summarize these results in the following proposition which follows from existence result in Proposition (2) .

Proposition 3. *Assume that the conditions of Proposition 2(I) hold, so that we have the existence of a CEM in the dual economy. As defined earlier, let* V^{TU*}_{BEM} , $V^{FU}_{BEM}(c)$ *and* $\bar{c}_{BEM}$ *be the value function of the unmatched traders, the value function of the unmatched farmers and the cutoff strategic rejection point in the economy prior to the introduction of the commodity exchange, and let the equivalent values after the introduction of the commodity exchange be* V^{TU*}_{DUAL},$V^{FU}_{DUAL}(c)$ *and* $\bar{c}_{DUAL}$. *Then (a)* $V^{TU*}_{BEM} > V^{TU*}_{DUAL}$; *(b) ,* $V^{FU}_{BEM}(c) < V^{FU}_{DUAL}(c)$ *for all c; and (c)* $\bar{c}_{BEM} < \bar{c}_{DUAL}$.

The different areas in Figure (9b) underscore the gains for different sets of farmers. It shows that farmers in the set $[0, \underline{c}_{DUAL}]$ move from the BEM to the CEM when it becomes available. Their benefit in terms of their value functions is the set $HEDG_1$. Those with cost parameters in the set $[\underline{c}_{DUAL}, \bar{c}_{BEM}]$ stayed in the BEM sector in both the pure BEM and the dual economy models. However the BEM is much better for the farmers in the dual economy, so they too obtain a benefit represented by the set $HG_1J\bar{c}_{BEM}$. Finally, there is the set of farmers with cost parameters in the set $[\bar{c}_{BEM},\bar{c}_{DUAL}]$ who were not served in the original pure BEM economy but are now served in the BEM sector of the dual economy. The benefit of those farmers is represented by the area in the triangle formed $\bar{c}_{BEM} J \bar{c}_{DUAL}$.

We now turn to our second measure of welfare: the aggregate mass of matches in the economy in every period. This measure of welfare, of course, ignores the costs of traders and the commodity exchange—which we discuss next. Earlier we indicated the aggregate supply in the pure BEM economy, Q^{BEM}, in equation (25) and that for the dual economy, Q^{DUAL}, in equation (38). It is easy to see that

$$Q^{DUAL} - Q^{BEM} = \left(\frac{1}{c^{max}}\right) \left\{ \underbrace{\int_0^{\underline{c}_{DUAL}} dc + m^* \int_{\underline{c}_{DUAL}}^{\bar{c}_{DUAL}} dc}_{\text{Matches in Dual}} - \underbrace{m^* \int_0^{\bar{c}_{BEM}} dc}_{\text{Matches in BEM}} \right\}.$$

After simple rearrangements, the equation above becomes

$$Q^{DUAL} - Q^{BEM} = \left(\frac{1}{c^{max}}\right) \left\{ \underbrace{\int_0^{\underline{c}_{DUAL}} (1-m^*) dc}_{\text{Direct mechanism}} + \underbrace{\int_{\bar{c}_{BEM}}^{\bar{c}_{DUAL}} m^* dc}_{\text{Indirect mechanism}} \right\} \tag{39}$$

The first term inside brackets captures what we refer to as the "direct" impact of the CEM. It represents the additional matches for farmers with cost parameters in the set $[0, \underline{c}_{DUAL}]$ who move from the BEM in the pure BEM economy to the CEM in the dual economy. A mass equal to $1-m^*$ of such farmers would be incurring post-harvest losses in the absence of the CEM. Equation (39) also highlights a second welfare channel, which we refer to as the "indirect" impact of the CEM. There is a set farmers with cost parameters in the set $[\bar{c}_{BEM}, \bar{c}_{DUAL}]$ who were not served by the BEM in the pure BEM economy, but that are now served in the dual economy. The probability of match, and hence expected output of each of such farmers is m^*.

Equation (39) provides a transparent measure of the new matches in the economy, but it does not incorporate the costs of the CEM (τ) or the trade costs of farmers (c). The next expression gives the total gains from the introduction in terms of values, defined by ΔW

$$\Delta W = \left(\frac{1}{c^{max}}\right) \left\{ \int_0^{\underline{c}_{DUAL}} (P-c)(1-m^*)\, dc + \int_{\bar{c}_{BEM}}^{\bar{c}_{DUAL}} (P-c)\, m^* dc - \int_0^{\underline{c}_{DUAL}} \tau dc \right\}.$$

This expression gives the flow of aggregate production value in the economy. To obtain the infinite discounted value of such flows, we have to compute their present value by dividing terms by $(1-\delta)$.[34] Here, the interpretation of τ matters for our discussion. If we interpret τ as a transfer from farmers to the CEM—as we treat the difference between P and $p(c)$ for the

[34]Since utility functions are linear in the value of goods, the welfare gain measured according to the aggregate value produced in this economy is directly related to the utilitarian welfare.

transactions between farmers and traders—, then the CEM will unambiguously increase the total value produced in the economy. However, if τ is a cost of production, then the impact of the CEM on the total value of production is ambiguous.[35]

4.6 Full or Complete Mandates and the Resilience of the traditional Bilateral Exchange Market

Related to our main result, Proposition (2), are two subtle observations, one with important policy implications. The first has to do with our result and the very definition of an equilibrium. Our proposition establishes the existence of an equilibrium whose form is a function of the parameter values of the model. The equilibrium is always unique and it is, again depending upon parameter values, either a pure BEM or else a dual economy. Suppose now that the government outlaws the BEM. This is what the government of Ethiopia has done for several crops which are important in the economy. If the BEM is banned, then of course the CEM will exist, but for a different set of farmers.

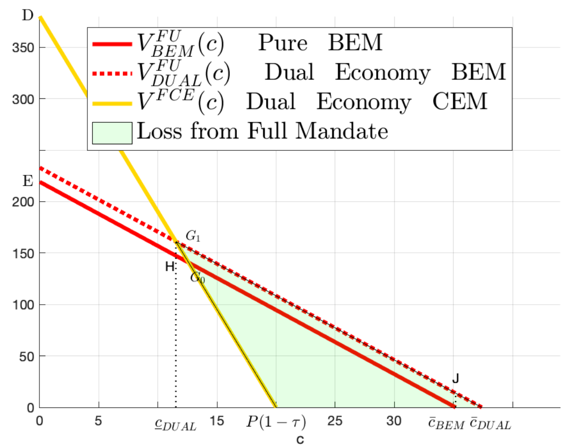
Indeed consider the diagram below. We draw the value functions of the farmers in the dual economy in the dual economy equilibrium. We could ask what would happen if instead of allowing a partial mandate (case of Ghana and Malawi) instead one would legislate a complete full mandate (case of Ethiopia). In the partial mandate the farmers served would be those of cost parameters from 0 to $\underline{c}_{DUAL}$ indicated in Figure (10) below. In the full mandate model, the CEM will be the only one which will operate, so the set of farmers would be $[0, P(1-\tau))$. In particular there will be a set of farmers, those in the set $(P(1-\tau), \bar{c}_{DUAL})$ that would have been served by the BEM in a dual market but are not being allowed to trade. In terms of welfare to farmers, those farmers with cost parameters in the set $(\underline{c}_{DUAL}, \bar{c}_{DUAL})$ would have gone to the BEM and obtained higher payoffs than in the full mandate model. The area shaded green is therefore the loss to farmers when the dual economy model is forced to be a CEM only model due to the full mandate legislation.

In Ethiopia, for the crops which have access to the commodity exchange and are subject to the full mandate, many have observed the existence of a "black market" where some farmers continue to illegally sell their produce through intermediaries, which is the equivalent of the BEM modeled here.[36] The result we have just described explains why such a black market

[35] In addition, to simplify the discussion, we assumed in the expression above that $\kappa = 0$. If $\kappa > 0$, we would have to take into account that in the CEM there is a larger mass of traders who spend κ in every period to search for farmers but who are not matched to any farmer. Specifically, we would have to add two additional terms: (i) one term related to the search costs incurred by traders who are no longer matched to low-cost farmers who sell to the CEM (i.e., $-\kappa\underline{c}_{DUAL}$), and (ii) another term related to the additional farmers who are matched to traders when there is a CEM (i.e., $(\bar{c}_{DUAL} - \bar{c}_{BEM})\kappa$).

[36] We spoke to a number of high ranking current and former officials at the Ethiopian Commodity Exchange. Some typical responses are the following: "A the Mercato (an open air market, perhaps Africa's biggest) ..

Figure 10: Full versus Partial Mandate



would exist. Our model also predicts that it is the high cost farmers who would be engaged in the black market. We underscore that, despite this legal requirement, the existence of dual markets is evidence of the desire of farmers to be in a BEM. If the laws were removed or relaxed we would see an even larger BEM than we actually do see in Ethiopia.

There is a second implication of our results. Our existence result shows that when the BEM is allowed, then regardless of how small the inefficiency of the CEM is, there will always be a traditional BEM in existence. One would conjecture that the smaller is the inefficiency (the tax rate τ) the smaller is the size of the BEM within the dual economy.

For the existence of the BEM, we know from our earlier propositions that the entry cost κ must be small. We will require the cost to be less than κ^{BEM}_{max} in equation (19) and also less than κ^{DUAL}_{max} in equation (37). The first bound, κ^{BEM}_{max}, is independent of τ. The second is a function of τ and, for a given κ, we require τ to be sufficiently large for the BEM to exist, and indeed this bound is given by equation (37) and equals

$$\hat{\tau} = \left(\frac{2c^{max}\sigma_2^2}{\delta P^2 \mu^T \tau^2 \sigma_1}\right)^{\frac{1}{2}} \kappa \tag{40}$$

An immediate corollary of our main Propositions is the following.

Corollary 1. *(a) Suppose $\kappa = 0$. Then regardless of how small is value of τ there will always be a BEM sector in the economy. (b) If $\kappa > 0$, then there will be a BEM sector in the economy so long as the commodity exchange tax rate τ exceeds the value $\hat{\tau}$ in equation (40).*

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people are opening selling grade 1 and grade 2 in the open market. They see the police and they run away. That black market was created. After a while they let this alone—i.e., the government. The government put controls on the (coffee) washing centers and aggregation areas. You arrest the woman, the next day the husband shows up. You arrest the husband and the son shows up. This is because it is reserved for export. The middle class wants the good coffee."

This corollary shows that, as long as the search cost κ is sufficiently small, there are always economic incentives for the existence of a BEM. We conclude this section by emphasizing that three ingredients are critical in the results from our model: (i) the search costs related to the probability of not forming a match, (ii) the heterogeneous cost parameters of farmers, and (iii) the commodity exchange market fees. The probability of not forming a match in the BEM generates incentives for some farmers to sell to the CEM. Heterogeneous costs, interacted with the CEM fees, determine which farmers sell to the BEM and which sell to the CEM. In the Online Appendix, we provide comparative statistics for different parameter values and show how the main results of the paper change when we change the parameters related to these three ingredients. In particular, we vary the values of c^{max}, τ, μ^F, and μ^T , and inspect the effects on $\bar{c}$, $\underline{c}$ and the gains from the CEM, as captured by equation (39).

5 Robustness Section

In our model, we employed different simplifying assumptions to focus on the essential features of the model. In this section, we discuss some of these assumptions, how they affect our results, and their relationships with our institutional context. Specifically, we cover five aspects of our model. First, the fact that traders match to at most one farmer. Second, the observability of the cost parameter c and its relationship to the search process of traders. Third, the different sources of price volatility implicit in our model. Fourth, the risk aversion of farmers. Lastly, the endogenous entry of traders.

One farmer to one trader match. In our model, one farmer matches to one trader. This formulation generates an important tradeoff: Traders compare the benefits of being immediately matched to a farmer against the expected value of being matched with farmers in the future. As a consequence, some high cost farmers who would generate a positive market surplus (i.e., farmers whose cost satisfy $p > c$) are strategically rejected by traders. The results of our model are the same to the results of an alternative model in which traders match to a finite, but multiple number of farmers. If this were say, n farmers per trader, we could think of considering the one trader with n farmers as being technically n traders, each having one trader to deal with.[37] This becomes equivalent to our one to one model.

[37] We highlight one technical issue here. We follow the vast majority of the literature in search and labor markets, which makes the assumption that that firms operate a constant returns to scale technology. Specifically, we assume that the technology that traders use to convert agricultural goods purchased from farmers into goods sold at retail markets is constant returns to scale. This assumption makes the negotiation of a trader with a farmer independent from her negotiation with another farmer. As discussed in detail in Elsby and Michaels (2013), with decreasing returns to scale the marginal worker generates less surplus than infra-marginal ones. Therefore, the rent that a firm obtains with a marginal worker is no longer independent from the rents that the firm obtain with others, which requires different solution methods.

If we assumed the extreme case in which traders can be matched to an unlimited number of farmers, the tradeoff mentioned above would disappear. In that case, traders would always be able to generate new offers to farmers in the future, without losing the opportunity of trading with the ones they have been currently matched with. As a consequence, there would be no strategic rejection. We believe that this extreme case, however, is inconsistent with our institutional setting. As described earlier in Section 2, traders in the agricultural markets of subsaharan Africa have limited capital and tend to be very small. They would not be able to handle the cash payments and the risk of larger volumes. Many traders either come into towns with their own small lorry, or else often carry the goods in a local private bus ("tro tro" or "Matatu"). That is, oftentimes, traders are even constrained by what they can physically carry themselves.

Search and observability of c. Traders often learn about trading opportunities by calling their relatives, friends and other traders. They also sometimes learn about these opportunities by asking around in the village during market days or by driving around trying to find farmers on the roadside of main roads. This search process comes with substantial uncertainty about the characteristics of farmers. As they search, traders have limited knowledge about the cost of the farmer that they might be able to find. Some of those farmers may need to bring their goods to the farm gate after hauling it over big hills, perhaps also a river or swamp. To capture this uncertainty in a parsimonious way, traders in our model imperfectly observe the farm specific cost c: They know the distribution of c in the village in which they are searching for farmers, but*, ex-ante*, they do not observe the specific value of c of the farmer that they might find.

In principle, one could extend our model to allow for traders who observe more information about the cost parameter c *ex-ante*. For example, suppose that there are two areas, call them A and B, with different distributions of the cost parameters, c. Area A could have primarily low c farmer types, while the other has high c types. Those with low c's will have higher potential gains to traders. If traders are to be indifferent between going the two areas A and B, the expected returns need to be the same. This would then mean that the places with higher potential gains should have lower probabilities of a match for the trader. Low c's would then imply lower probability of finding a match. (This is similar to the posted wages models— see, e.g., Rogerson, Shimer, and Wright (2005).)[38]

[38] In this less parsimonious version of the model, we would have to characterize several additional features in the model to prove existence and uniqueness of equilibria. For example, we would have to model how traders choose between sub-villages, and those traders would have to be indifferent between going to sub-villages where farmers have a low average c versus a village where farmers have a high average c. In equilibrium, the expected return from searching in these different sub-villages would have to equalize.

Aggregate vs farm-level price volatility. Conceptually, we think of two sources of price volatility in our model: (i) one source related to the aggregate or final price of produce in retail markets P; (ii) another source related to the price that farmers receive for their produce, which can be $p(c)$ or 0 depending on whether or not farmers sell their goods to traders. We modeled the second source and not the first. Of course, we could have gone further and attempted to model randomness in the final or "Accra" price P itself. One could ask: if we had gone the route of modeling the uncertainty in the aggregate or macro price P, what would happen? In that case, we would have to be specific about the timing and who has what price information at the time of the match and the bargaining. If they (the trader and the farmer) both have no information on P before trading, then they would replace the price P with the expected price $E(P)$, and the analysis would follow very similarly to what we have modelled. However, if one side has better information than the other then we obtain a much more complicated search problem where the uninformed needs to infer the information of the informed.[39] Here, we abstracted from aggregate price volatility as to focus on the impact that a commodity exchange has on the ability of farmers to find traders (the question of post-harvest losses).[40]

Risk Aversion. Let us now consider the introduction of risk aversion on the part of farmers. In that case, we would replace the linear utility functions of farmers with some concave function u. In this connection, there is the Nash Bargaining to consider. In a static model, that would change the bargaining weights—see for example Li, Sun, Yan, and Yin (2015), who study the types of utility functions which move the Nash Bargaining weights one way or the other. In our parametric model, since we allow the bargaining weights to be a parameter in principle determined by the data, we do not believe that this modification would not change the essence of our results. In addition, in comparison to a linear utility function, a concave utility function on the part of farmers would increase the preference for the stability in prices afforded by the CEM, relative to the BEM. The precise nature of the increased preference would of course depend upon the concavity of the new utility function.

Endogenous entry of traders. As discussed in Section 2, there are several factors limiting the entry of traders in agricultural markets in Subsaharan Africa, among them, the existence of market queens or capital constraints. Based on this observation, we assumed a fixed mass of traders in our model. As such, our modeling approach contrasts with the common assumption of free entry of firms adopted in search models of labor markets. What would happen to the

[39] See, e.g., Hildebrandt, Nyarko, Romagnoli, and Soldani (2015) which models this situation—in that paper the trader has better information than farmers (at least most of them).

[40] Empirically, the few papers evaluating the impact of commodity exchange markets based on the Ethiopian case find weak to no evidence of an impact on aggregate price volatilities. Tadesse and Guttormsen (2011) study the impact of the Ethiopian commodity exchange (ECX) on price volatility of maize and teff and Hernandez, Lemma, Rashid, et al. (2015) the impact of ECX on price of coffee.

existence of strategic rejection and the dual markets if there were free entry of traders? If we assumed free entry of traders with a zero profit condition, that would drive V^{TU} to zero, since a positive V^{TU} would lead to an inflow of traders to the region. Because $\bar{c} = P - (1-\delta)V^{TU}$, the free entry of traders would imply $\bar{c} = P$ and remove any strategic rejection. We think that this extreme assumption, however, is inconsistent with our observations about agricultural markets in Subsaharan Africa.[41]

As for the existence of dual markets, Proposition (2) showed that, as long as $\tau < \tau_2$, some farmers would choose to sell to the BEM ($\underline{c} > 0$), for any equilibrium value of V^{TU} (including zero). Therefore, the free entry of traders would not affect the existence of dual markets. Intuitively, the existence of a dual market hinges on the negotiated price of an agricultural good in the BEM being sufficiently high. In that case, from a farmers' perspective, there exists a tradeoff between selling their produce to a trader at a high-price, but at the risk of incurring post-harvest losses, or selling their produce to the commodity exchange at a lower price, but at no risk of incurring post-harvest losses.

6 Conclusion

In this paper, we formulated a search model to study the introduction of a commodity exchange market in a rural village where traders and farmers trade in a decentralized market. We show that search frictions in the bilateral exchange market generate two types of economic disadvantages. First, there are some farmer-trader matches which do not occur simply because farmers and traders do not find each other. Second, there are matches that do not take place because traders strategically reject the farmers that they are matched to because their costs are too high and it is in the trader's interest to re-sample the market. By introducing a commodity exchange market in this environment, we found that this market institution eliminates many of the economic disadvantages of the bilateral trading environment. Curiously, we found that despite the advantages of the commodity exchange, there could still be dual markets where the commodity exchange co-exists with the bilateral trade environment. This occurs when the commodity exchange charges high transaction fees and high-cost farmers find it profitable to stay in the bilateral trading environment.

One of the implications of the theoretical model is that many of the traders who were in existence in the bilateral trading environment will go out of business with the introduction of the commodity exchange. This is because the commodity exchange is able to provide intermediation much better than the traders. The commodity market, by creating a centralized market, is

[41] Alternatively, one could imagine a model in which there is a supply curve of traders, where the mass of traders n_t^T is a function of V^{TU}. In this alternative model, because the mass of traders is a function of V^{TU}, the fixed-point strategy that we applied to solve for the equilibrium in our model would have to be reformulated.

able to eliminate the lost farmer-trader matches. This was seen upon the introduction of the Ethiopian Commodity Exchange about a decade ago. In Ethiopia, these traders in the bilateral model were called Akrabis. For the most part, the Akrabis were wiped out. We suspect the same will be true in Ghana's case, as well as other African countries as they implement commodity exchange markets of their own. If the commodity market fees are too high, however, there may be high-cost farmers still in the bilateral trading environment.

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Online Appendix for "From Bilateral to Decentralized Markets: A Search Model for Commodity Exchanges in Africa"

Yaw Nyarko and Heitor S. Pellegrina

A Additional discussion about μ^T and μ^F

Throughout the paper, we have assumed that the matching probabilities, μ^F and μ^T were exogenous. How serious an assumption is this, and what are the implications of relaxing this assumption? In this section, we present additional discussion about the formation of μ^F and μ^T. Part of the reason for not explicitly modeling these parameters endogenously is that there are not too many theoretical models of these match probabilities that are used in the literature. There is a Cobb-Douglas type matching function which is popular. However that matching function does not have, as far as we are aware, explicit theoretical foundations.

So, where would the exogeneity of μ^F and μ^T matter? First, from our assumption that both farmers and traders are small, they will take the match probabilities μ^F and μ^T fixed when forming their individual optimization decisions and hence their value functions will be just as described in (1) - (5). The expressions for the pure BEM model would therefore hold, as would the expressions for the dual economy. The equations presented in this paper describing the equilibrium therefore remain the same.

The potential concerns with exogeneity of the match probabilities occur at the following places.

1. Existence of equilibrium: It is possible that there is yet some other sets of equations which determine the match probabilities which impose additional constraints on the model in a manner tied to the value functions in such a way that an equilibrium no longer exists. If we do have existence of an equilibrium, however, the functional forms presented in the paper will continue to hold. Care would have to be taken when doing comparative statics as a change in one parameter, say β, would also impact the match probabilities if they are endogenous. The main result of the paper, however, on the possibility of a dual economy would of course continue to hold despite this issue with comparative statics.

2. Introduction of the CEM: We argued above that the value functions and expressions continue to hold for the pure BEM and for the dual economy. One however may be worried that the match probabilities in the pure BEM will differ from the match probabilities in the dual economy. That would affect our comparison results in section on the Welfare and Gains from trade which compared the pure BEM model with the dual economy model. The issue there would be the valid concern that the match probabilities, which we have assumed fixed, indeed change when going from one environment to the next. With an explicit matching technology in one versus the other we could of course compute each economy with its own matching function. In the absence of that, one may ask how much of an issue is it to assume that the matching probabilities remain the same? We believe it provides a ballpark analysis of the impacts of the introduction of a commodity exchange. With the introduction of the commodity exchange there will be a reduction of the number of farmers available to the BEM sector. However, with profit margins falling, there will also be a reduction in the number of traders in the BEM. Under the standard matching functions used in the literature, proportional reductions in the numbers of both sides of the market results in the match probabilities remaining the same. That is what we assume here.

B Additional discussion on relationship between prices and costs

Prices are increasing in c in our model. Is this a good description of the reality of farmers in our developing country context? Profits (the difference between price and cost) will of course generally be lower for high cost farmers since the gains from trade will be lower.

We think of the parameter c as the reservation level of the farmer. Those with high values of c would need high prices before they could be induced to enter into the market. It is therefore not surprising that we obtain in equilibrium high costs associated with high prices. Further, we model the bargaining between the farmer and the trader in the bilateral exchange economy as following the Nash Bargaining process. This implies that the farmer and trader set the prices $p(c)$ so as to split the gains from trade. Those gains are defined by their reservation values – the cost c for farmers and the external price P for traders. The price will lie between these two bounds. This has the implication that farmers with high costs, i.e., those with higher reservation values will obtain higher prices in the equilibrium with Nash bargaining. Search complicates the above, but in a static model the above would be straightforward.

One could object with our modeling above by saying that those with high costs should somehow be those with weaker bargaining power who therefore obtain lower prices from traders.[42] That is, prices should be a decreasing function of the cost parameter c. It is easy to show that prices are everywhere decreasing in costs then there will be a farmer-trader pairs who have gains from trade but do not trade because equilibrium price is less than the cost.[43] More generally, this argument shows that in any reasonable model there will be portions of the price function which are upward sloping as we have here in our standard bargaining model.

In a more general model, beyond the scope of this current paper, one could imagine a situation where the bargaining power could vary by c. The bargaining power parameters are typically derived from the relative rates of impatience of the players in the bargaining game, here the farmer and the trader. For example, some farmers may be more patient because have better storage facilities or access to warehouses. For the communities studied in this paper, there is almost no formal storage anywhere, and to the extent that there is storage it is a function of the region of the country the farmer lives in. Farmers in the northern part of the country where it is drier have good storage of maize in their backyards (sheds outside their homes). Those in the south, where it is more humid, are unable to store for long without getting the crops infested with insects or disease (aflatoxins with maize). That is the motivation in this paper for our following the standard literature in modeling the bargaining parameter as constant, leaving the case of variable bargaining power for future research. We do believe, however, that even in a more general model with bargaining power dependent on cost parameters we believe that the main conclusions of this paper will continue to hold. This is because the main results follow from variability of profit levels (not prices) as a function of the cost parameter which is usually downward sloping here, and it seems unlikely that that would change significantly even with bargaining power a decreasing function of c.

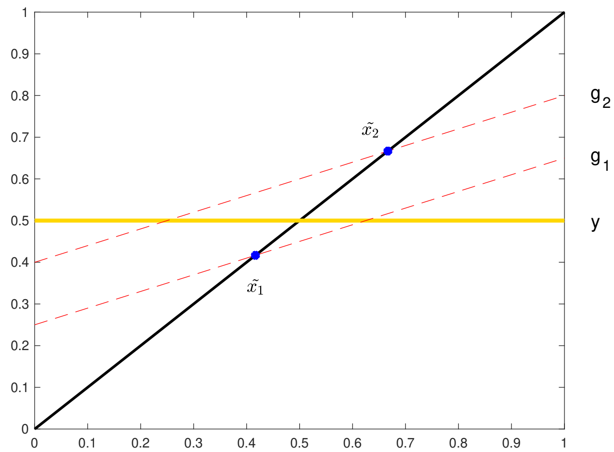
C The Proofs

This section presents proofs for the propositions presented in the paper.

[42]In our model that would mean that the bargaining share going to the farmer (ϕ in our model) is decreasing in c. Not only that, but the effect of c on ϕ should so dominate the reservation effect mentioned earlier (prices must exceed c for farmers to want to trade) that on net the prices are falling in c.

[43]Think of a graph with c on the horizontal and prices on the vertical. Assume, for the sake of argument, that the price function $p(c)$ is downward sloping everywhere from some initial value $p(0)$ at $c = 0$. Then it will intersect the 45-degree line at some $c_{45}>0$. For all values of c greater than that value c_{45} trades would not be occur (since $p(c) < c$ so farmers do not want to trade) even though there are gains from trade ($P > c$).

Figure A.1: Proof for the existence of $V^{TM}(c)$



C.1 Proof of Claim 1

Proof. From easy algebra one can show that

$$\sigma_1 = \phi(1-\delta) + \delta\beta + \beta\delta\phi + \delta\mu^F\phi + 1 - \delta > 0,$$

and

$$\sigma_2 = \phi(1-\delta)(1-\mu^F) + \phi\beta\delta + \beta\delta + (1-\delta) > 0.$$

□

C.2 Proof of Claim 2

Proof. To see why Claim 2 is true, we re-write the terms from the claim above in terms of another claim presented below to make the proof easier to see. By setting $y = V^{TU}$, $x = V^{TM}(c)$, $\tilde{x} = \tilde{V}^{TM}(c)$ and $g(x) = V^{TM}_{RHS}(c)$ as in Claim (7), we prove Claim (2). One can check that, as a function of $V^{TM}(c)$, $V^{TM}_{RHS}(c)$ is linear in $V^{TM}(c)$ with slope < 1 (and possibly negative) and intercept > 0 and, therefore, it satisfies the conditions of claim (7). □

Before we present the next claim, which we use to prove Claim 2, we illustrate its intuition in Figure A.1. If $g(x) = g_1(x)$, $g(x)$ has a fixed point $\tilde{x_1}$. For $y > 0$, $\max\{g(x), y\}$ is the function which is equal to y for x between 0 and around 0.6 and is equal to g_1 for all values higher than x, and has a unique fixed point equal to $x^* = \max\{\tilde{x}, y\} = y$. Alternatively, if $g(x) = g_2(x)$ then g has a fixed point $\tilde{x_2}$ and for $y > 0$, $\max\{g(x), y\}$ is the function which is equal to y for x between 0 and around 0.25 and is equal to g_2 for all higher x, and has a unique fixed point equal to $x^* = \max\{\tilde{x_2}, y\} = \tilde{x_2}$.

Claim 7. Let $g(x)$ be a real valued function on $[0, \infty)$. Suppose that $g(x)$ is linear in x with slope < 1 and that $g(0) > 0$. Then (i) $g(x)$ has a unique fixed point $\tilde{x}$, so that $\tilde{x} = g(\tilde{x})$; (ii) for any fixed $y > 0$, the function of x defined by $x = \max\{g(x), y\}$ has a unique fixed point equal to $x^* = max\{\tilde{x}, y\}$ (i.e., if x^* is as just defined, then $x^* = \max\{g(x^*), y\}$); and (iii) the fixed point in (ii) is unique.

Proof. (i) Since $g(0) > 0$, $g(x)$ is linear and the slope of $g(x)$ is strictly less than one, it is easy to see that $g(x)$ will have a unique fixed point $\tilde{x} > 0$. Further $g(x)$ will lie above the 45 degree line for $x < \tilde{x}$ (i.e., $g(x) > x$) and below it for $x > \tilde{x}$ (i.e., $g(x) < x$). (ii) First, suppose that $\tilde{x} \leq y$. Then, we have $x^* = \max\{\tilde{x}, y\} = y$ and, from the proof of (i) above, $g(y) \leq y$. Using $x^* = y$, $g(y) < y$ and $x = \max\{g(x), y)\}$, we have $x^* = y = \max\{g(y), y\} = \max\{g(x^*), y\}$ which shows that x^* is the solution to the fixed point in $x = \max\{g(x), y\}$ when $\tilde{x} \leq y$. Second, suppose that $\tilde{x} > y$. Then, we have $x^* = \max\{\tilde{x}, y\} = \tilde{x}$. Using $x^* = \tilde{x}$, $\tilde{x} = g(\tilde{x})$ and $x = \max\{g(x), y)\}$, we have $x^* = \tilde{x} = \max\{\tilde{x}, y\} = \max\{g(\tilde{x}), y\} = \max\{g(\max\{\tilde{x}, y\}), y\} = \max\{g(x^*), y\}$, which shows that x^* is the solution to the fixed point in $x = \max\{g(x), y\}$ when $\tilde{x} \le y$. (iii) We now show that the fixed point x^* is unique. Suppose, on the contrary, there is another fixed point x^{**} so that $x^{**} = \max\{g(x^{**}), y\}$. If $g(x^{**}) \ge y$, then $x^{**} = \max\{g(x^{**}), y\} = g(x^{**})$ so x^{**} is a fixed point of $g(x)$ so $x^{**} = \tilde{x}$. Hence $x^{**} = \max\{g(x^{**}), y\} = \max\{g(\tilde{x}), y\} = \max\{\tilde{x}, y\} = x^*$. Alternatively, if $g(x^{**}) < y$, then $x^{**} = \max\{g(x^{**}), y\} = y$. This in turn means that $g(y) = g(x^{**}) < y$. Then from part (i) of the proof of this claim $y > \tilde{x}$. Hence $x^{**} = \max\{g(x^{**}), y\} = y = \max\{\tilde{x}, y\} = x^*$. □

C.3 Proof of Proposition 1

Proof. First, suppose that $\kappa < \kappa^{BEM}_{max}$. We are looking for values of V^{TU} which are solutions to the equation $V^{TU} = V^{TU}_{RHS}$. We saw in the claim above that at $V^{TU} = 0$, we have $V^{TU} < V^{TU}_{RHS}$ when $\kappa < \kappa^{BEM}_{max}$ and $V^{TU} > V^{TU}_{RHS}$ at $V^{TU} = P/(1-\delta)$. From the intermediate value theorem the function $V^{TU} - V^{TU}_{RHS}$ has a solution, or V^{TU} is equal to zero. Since the V^{TU}_{RHS} function is convex, it is easy to show that V^{TU} can only be a solution between 0 and $P/(1-\delta)$. Second, suppose $\kappa > \kappa^{BEM}_{max}$. Then the earlier arguments show that $V^{TU} = V^{TU}_{RHS}$ at both $V^{TU} = 0$ and at $V^{TU} = P/(1-\delta)$. Since V^{TU}_{RHS} is convex it is easy to see that there cannot be a $V^{TU} > 0$ such that $V^{TU} = V^{TU}_{RHS}$. Finally, when $V^{TU} = 0$, equation (4) can only be satisfied with $V^{TM} = 0$ and hence $V^{FM} = V^{FU} = 0$ which can only occur if there are no traders making visits to farms. □

C.4 Proof of Claim 4

Proof. $V^{FU}_{slope}/V^{FCE}_{slope} = \frac{\mu^F \phi}{\sigma_1}$. Recall from Claim (1) above that $\sigma_2 = \sigma_1 - \mu^F \phi > 0$ and so $\mu^F \phi < \sigma_1$ and hence $\mu^F \phi \ / \ \sigma_1 < 1$. Hence $V^{FU}_{slope}/V^{FCE}_{slope} < 1$. □

C.5 Proof of Proposition 2

Proof. As mentioned earlier, it suffices to find an equilibrium value for V^{TU}, since with that we can re-trace our steps and obtain equilibrium values of the other value functions. (I) (a) Suppose we have the first condition in part (I): $V^{TU}_{\underline{c}} > 0$ and $\overline{V}^{TU}_{\underline{c}} > V^{TU}_{\underline{c}}$. Figure 7a will be helpful in the proof (the figure is drawn from the explicit parameter values [c^{max}, P, β, δ, μ^F, μ^T, ϕ, τ] = [40, 40, 0.1, 0.5, 0.1, 0.9, 0.5, 2, 0.3]). Since $\overline{V}^{TU}_{\underline{c}} > V^{TU}_{\underline{c}}$, both $V^{TU}_{RHS,BEM}$ and $V^{TU}_{RHS,DUAL}$ are above the 45 degree line at $\overline{V}^{TU}_{\underline{c}}$. They are both below the 45 degree line at some point bigger than $\overline{V}^{TU}_{\underline{c}}$($V^{TU}_{RHS,BEM}$ at $V^{TU} = \frac{P}{1-\delta}$ and $V^{TU}_{RHS,Dual}$ at $V^{TU} = \frac{P\tau}{1-\delta}$). Hence from the intermediate value theorem they each have a fixed point at some $V^{TU} > V^{TU}_{\underline{c}}$. From Proposition 1 and Claim 6, both $V^{TU}_{RHS,BEM}$ and $V^{TU}_{RHS,DUAL}$ have unique fixed points on their respective domains. In particular, neither $V^{TU}_{RHS,BEM}$ nor $V^{TU}_{RHS,DUAL}$ has a fixed point on [0, $V^{TU}_{\underline{c}}$). On V^{TU} in ($V^{TU}_{\underline{c}}$, $\frac{P\tau}{1-\delta}$),we are in the Case I described earlier so the correct value function to use is $V^{TU}_{RHS,DUAL}$, and we just argued that there is a fixed point of $V^{TU}_{RHS,DUAL}$ on ($V^{TU}_{\underline{c}}$, $\frac{P\tau}{1-\delta}$). This shows that a fixed point exists. The fixed point will be unique since we just argued that $V^{TU}_{RHS,BEM}$ has no fixed point on [0, $V^{TU}_{\underline{c}}$) and $V^{TU}_{RHS,BEM}$ is the appropriate value function to use on [0, $V^{TU}_{\underline{c}}$). (b) Suppose we have the second condition in part (I): $V^{TU}_{\underline{c}} < 0$. In this case the dual value function $V^{TU}_{RHS,DUAL}$ is the operative one for all values of V^{TU}. The proposition therefore follows from part 4 of Claim 6 .

(II) Figure 7b will be helpful in the proof (the figure is obtained from parameter values parameter values [c^{max}, P, β, δ, κ, μ^F, μ^T, ϕ, τ] = [40, 40, 0.1, 0.5, 0.005, 0.9, 0.5, 2, 0.4]). The proof is analogous to that in (I). This can be omitted, but we include it here for the sake of completeness. Since $\overline{V}^{TU}_{\underline{c}} < V^{TU}_{\underline{c}}$, both $V^{TU}_{RHS,BEM}$ and $V^{TU}_{RHS,DUAL}$ are below the 45 degree line at $\overline{V}^{TU}_{\underline{c}}$. They are both above the 45 degree line at $V^{TU}=0$. Hence form the intermediate value theorem they each have a fixed point at some V^{TU}in $(0, V_{\underline{c}}^{TU})$. From Proposition 1 and Claim 6, both $V_{RHS,BEM}^{TU}$ and $V_{RHS,DUAL}^{TU}$ have unique fixed points on their respective domains. In particular, neither $V_{RHS,BEM}^{TU}$ nor $V_{RHS,DUAL}^{TU}$ has a fixed point on $(V_{\underline{c}}^{TU}, \frac{P\tau}{1-\delta})$. On V^{TU} in $(0, V_{\underline{c}}^{TU})$ we are in the Case II described earlier so the correct value function to use is $V_{RHS,BEM}^{TU}$, and we just argued that there is a fixed point of $V_{RHS,BEM}^{TU}$ on $(0, V_{\underline{c}}^{TU})$. This shows that a fixed point exists. The fixed point will be unique because we just argued that $V_{RHS,DUAL}^{TU}$ has no fixed point on $(V_{\underline{c}}^{TU}, \frac{P\tau}{1-\delta})$ and $V_{RHS,DUAL}^{TU}$ is the appropriate value function to use on $(V_{\underline{c}}^{TU}, \frac{P\tau}{1-\delta}))$.

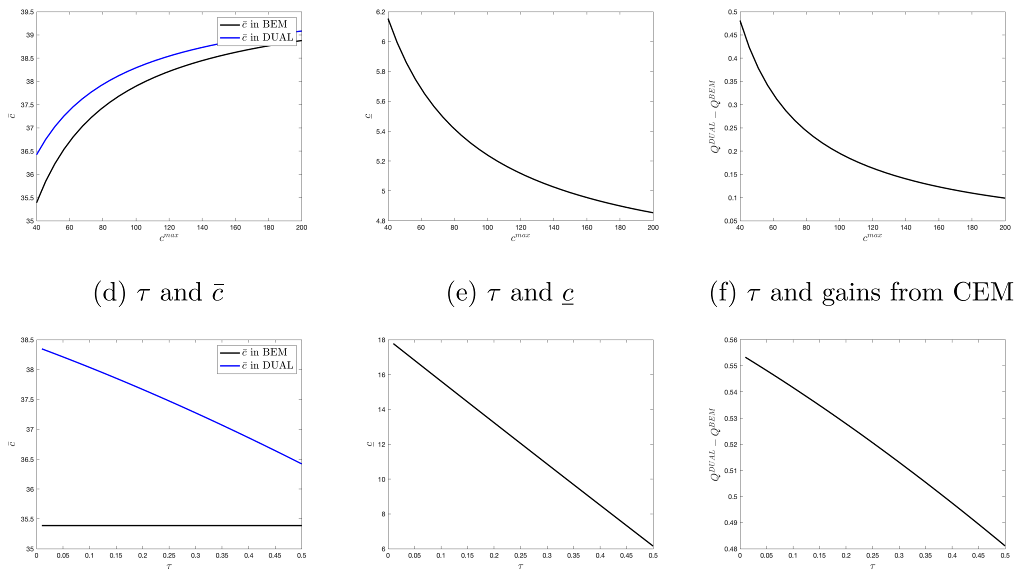
When $V_{\underline{c}}^{TU} < 0$, then for all V^{TU}, as is easily seen from Figure 5a or Claim 5, $0 \leq \underline{c} \leq \bar{c}$ and so $V_{RHS,DUAL}^{TU}$ is the appropriate value function to use. Claim (6) then implies the existence of a unique equilibrium which is a dual equilibrium. □

D Comparative Statics

This section shows how the selection of farmers into BEM and CEM and the gains from the introduction of CEM (computed based on equation (38)) vary with different parameter values. In particular, we inspect the results from our model for $\kappa = 0$, which ensures the existence of the BEM (see Corollary 1). In what follows, we use the following set of parameters as our benchmark [c^{max}, P, β, δ, κ, μ^F, μ^T, ϕ, τ] = [40, 40, 0.5, 0.952, 0, 0.2, 0.8, 1, 0.5]. We solve the equilibrium of the model, with and without farmers' option to sell their produce to the CEM, using different values of μ^F, μ^T, and c^{max}, holding all the other parameters fixed at their benchmark values.

Panel (a) in Figure (A.2) presents the effects of increasing the overall costs of farmers in the economy. As we increase the average cost, $\bar{c}$ increases both in the BEM and in the CEM. This occurs because traders have fewer farmers with production costs below $\bar{c}$ with whom they can trade and produce positive market surplus. For similar reasons, $\underline{c}$ falls also with an increase in c^{max}. The positive impact of the CEM drops with an increase in farmers cost. This occurs because low-cost farmers who are the ones who tend to self-select into the CEM receive better prices with traders when average costs are higher.

Figure A.2: Alternative Parameter Values (c^{max} and τ)



(a) c^{max} and $\bar{c}$ (b) c^{max} and $\underline{c}$ (c) c^{max} and gains from CEM

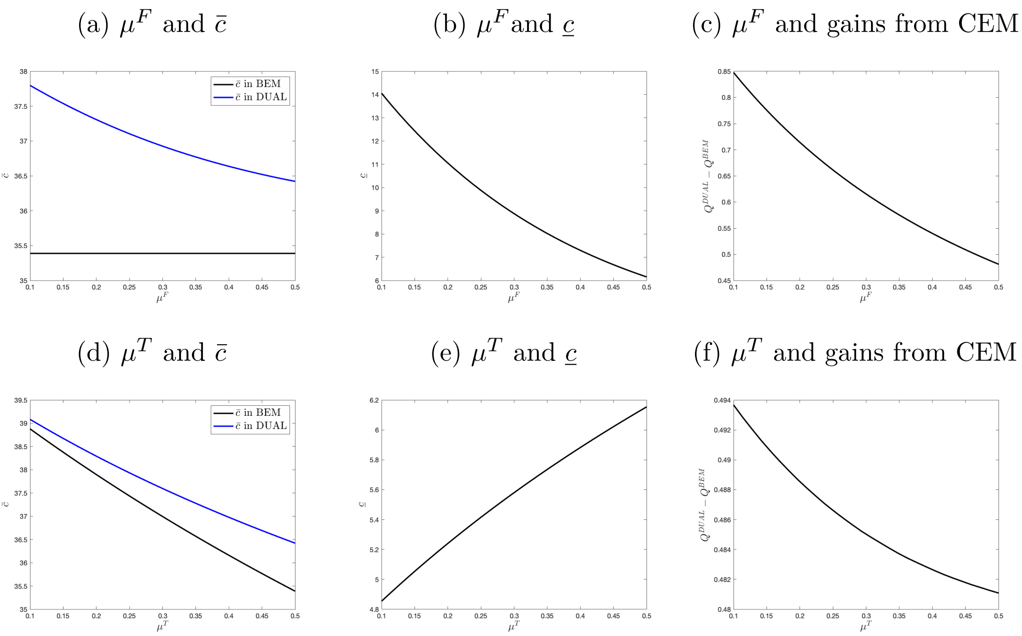
(d) τ and $\bar{c}$ (e) τ and $\underline{c}$ (f) τ and gains from CEM

Notes: These figures show simulations of the model for different parameter values related to farmers' cost (c^{max}) and to the fees charged by the commodity exchange market (τ). In particular, panel (a), (b) and (c) show the effects of changing c^{max} and panels (d), (e) and (f) the effects of changing τ.

Panels (b), (c) and (d) in Figure (A.2)document the effects of changing τ, the fee charged by the commodity exchange market. It shows that an increase in τ leads to a reduction in $\bar{c}$ of the CEM. This occurs because a larger proportion of farmers choose to sell their produce in the BEM, which is captured by a drop in $\underline{c}$, shown in Panel (e). This drop in $\underline{c}$ is beneficial for traders, who have better access to low-cost farmers and therefore tend to reject a larger share of high-cost farmers. As fewer farmers choose to stay in the CEM with higher τ, the impact of the CEM on the overall production also drops.

Lastly, Figure A.3 presents the effects of changing the matching probabilities. Intuitions for the results obtained are analogous to the ones discussed for Figure A.2.

Figure A.3: Alternative Parameter Values (μ^T and μ^F)



Notes: These figures show simulations of the model for different parameter values related to the probability that farmers find a trader (μ^F) and the probability that traders find a farm (μ^T). In particular, panel (a), (b) and (c) show the effects of changing μ^F and panels (d), (e) and (f) the effects of changing μ^T.

E Examples of Receipts from the GCX

Figure A.4: Example of Actual Sales of two different farmers in the Ghana Commodity Exchange (GCX)

(a) Example 1

Ghana Commodity Exchange

Trade Date: 13/08/2020

Net Obligation Report

Receipt No: 0000000602

Client Name: [redacted] . A[redacted] **Client ID:** [redacted] **VAT:** 3598 **TIN:** [redacted]

Member Name: [redacted] (B) **Member ID:** [redacted] **VAT:** 1212 **TIN:** [redacted]

Sell Details

Trade ID	WHR ID	Symbol	Quantity	Price	Actual Weight	Trade Value	Adjusted Trade Value	Grading	Weighing	Re-bagging	Handling	Storage	Fumigation	Elec. WHRing	Trading	Regulatory	CD	MLA	Cleaning	Drying	Total Fees	Settled	Deferred	Settlement Date
580	0000000829	GKUWM3	5.8444	1200.00	5844.39	7013.28	7013.27	234.00	0.00	0.00	175.50	280.80	0.00	11.69	63.82	14.03	0.00	0.00	351.00	0.00	1130.84	5882.43	0	13/08/2020
																		Total:	5882.43					

Net: 5882.43

(b) Example 2

28/09/2020

Ghana Commodity Exchange

Trade Date: 25/09/2020

Net Obligation Report

Receipt No: 0000000798

Client Name: [redacted] . **Client ID:** [redacted] **VAT:** 837399393 **TIN:** [redacted]

Member Name: [redacted] LTD [redacted] **Member ID:** [redacted] **VAT:** 1821 **TIN:** [redacted]

Sell Details

Trade ID	WHR ID	Symbol	Quantity	Price	Actual Weight	Trade Value	Adjusted Trade Value	Grading	Weighing	Re-bagging	Handling	Storage	Fumigation	Elec. WHRing	Trading	Regulatory	CD	MLA	Cleaning	Drying	Total Fees	Settled	Deferred	Settlement Date
726	0000000802	GSAYM1	1.0000	1300.00	983.03	1300.00	1277.94	40.00	0.00	0.00	30.00	80.00	0.00	2.00	11.83	2.60	0.00	0.00	0.00	0.00	166.43	1111.51	0.00	28/09/2020
																		Total:	1111.51					

Net: 1111.51

Notes: These figures show actual sales of farmers to the Ghana Commodity Exchange (GCX). The first farmer paid approximately 16% (= 1130 / 7013 * 100) in fees relative to total sales to the GCX and the second one paid 13% (= 166 / 1277 * 100). For privacy reason, we have blocked the client and member names in red.

Figure A.5: Example of a Receipt received by a Farmer in the Ghana Commodity Exchange (GCX)

06/04/2020

GCX

Ghana Commodity Exchange

GOODS RECEIVED NOTE

Tamale

0000000496

NT-0091-5856

Phone No:

Crop Owner

Member/Client

[redacted]

Crop Detail

Production Area	Commodity	Commodity Class	Origin	Harvest Year
N/A	Maize	White Maize	Tamale	2020

Grade	Symbol	No. of Bags	Net Weight	Lot (MT)
Tamale White Maize 3	GTAWM3	216	10616.7240	10.6167

Crop Location

Tracking Code	Warehouse	Shed	Stack
WHRG1238	Tamale	TAS1	TAS1-1

Crop History

Arrival Timestamp:	06/04/2020 11:14:00
Date Sample Taken:	06/04/2020 09:40:00
Graded Timestamp:	06/04/2020 10:02:00
Deposited Timestamp:	06/04/2020

Mandatory Fees

Service	Fee (GHC)	Service	Fee (GHC)	Service	Fee (GHC)
Grading, Weighing & Rebagging (Per Bag)	2.00	Moisture Loss (%Volume)	0.00	Handling (Per Bag)	1.50
Weighing (Per Bag)	0.00	Receipting Fee (Per MT)	2.00	Trading (% Value)	0.91
Central Depository (% Value)	0.00	Storage (Bag/Month)	0.80	Regulatory Fee (%Value)	0.20

Optional Service Fees

Service	Fee (GHC)	Service	Fee (GHC)	Service	Fee (GHC)
Drying (Per Bag)	6.00	Re-Bagging (Per Bag)	0.00	Fumigation (Per MT)	10.00
Cleaning (Per Bag)	3.00				

Received By: [redacted]

Signature: [redacted]

Date: 06-04-2020

Staff Name: [redacted]

Signature: [redacted]

Date: 06-04-2020

Notes: This figure shows a receipt given to the farmer for depositing his produce at the commodity exchange market. It includes the price of mandatory and optinal service fees. It shows that the produce was deposited on 06/04/2020. For privacy reason, we have blocked the names and signatures in red.

F Appendix Tables

Table A.1: General Characteristics of Market Transactions in Selected African Countries (%)

	Ethiopia 2015 (1)	Tanzania 2010 (2)	Malawi 2010 (3)
a. Main mode of transportation			
- Foot / Bike / Draft Animal	91	80	49
- Truck / Bus / Minibus	4	18	18
- Other	5	2	33
b. Main storage method			
- Traditional (Bags, Unprotected Piles,...)	79	97	80
- Modern (Metallic Silos, ...)	1	1	20
- Other	20	2	0
c. Main reason for storage			
- Household consumption	82	73	87
- To sell at a higher price	6	3	8
- Other	12	24	5
d. Number of transactions while selling			
- One transaction	70	84	78
- Two transactions	20	6	10
- Three transactions	6	3	5
- Other	4	7	7
e. Any other buyer in addition to your main buyer			
- Yes	-	-	2
- No	-	-	98
f. Period when most crop was sold			
- Within 4 months (high season)	96	70	87
- Other	4	30	13

Notes: Data comes from Living Standard Measurement Study (LSMS) organized by the World Bank.

Table A.2: Commodity Exchanges in Africa

	Country	Founded	Status	Commodities	Contracts
JSE	South Africa	1995	Operational	Maize, wheat, soy and sunflower	Futures
ACE	Malawi	2004	Operational	Rice, wheat, beans, ground nuts and peas	Forward
ECX	Ethiopia	2008	Operational	Coffee, sesame, beans, wheat, maize	Spot
ZIMACE	Zimbabwe	1994	Closed in 2001	Maize, wheat, soy	Spot and forward
ZAMACE	Zambia	2007	Closed in 2012	Maize, wheat, soy, sunflower	Spot and forward
GCX	Ghana	2018	Implementation	Maize	Spot and forward
KCX	Kenya	1997	Operational	Main niche is perishable commodities	-
Bourse Africa Ltd.	Mauritius	2010	Operational	Gold, silver and crude oil	Futures and CFDs
TCX	Tanzania	2015	Project	Cashew nuts, coffee, cotton and rice	-

Notes: Bourse Africa Limited was previously named GBOT.